Homework Solution
Chapter 5

Pb#2.

A particle is described by the wavefunction

$$\psi(x) = \begin{cases} A\cos\left(\frac{2\pi}{L}x\right) & -\frac{L}{4} \le x \le \frac{L}{4} \\ 0 & \text{otherwise} \end{cases}$$

a)

$$\int_{-\frac{L}{4}}^{\frac{L}{4}} \psi_{(x)}^2\, dx = 1 = A^2 \int_{-\frac{L}{4}}^{\frac{L}{4}} \cos^2\left(\frac{2\pi}{L}x\right) dx$$

$$2\cos^2\theta = 1 + \cos 2\theta$$

$$1 = \frac{A^2}{2}\int_{-\frac{L}{4}}^{\frac{L}{4}} \left[1 + \cos\left(2\frac{\pi}{L}x\right)\right] dx = \frac{A^2}{2}\left[x\Big|_{-\frac{L}{4}}^{\frac{L}{4}} + \frac{L}{2\pi}\sin\left(\frac{2\pi}{L}x\right)\Big|_{-\frac{L}{4}}^{\frac{L}{4}} \right]$$

$$= \frac{A^2}{2}\left[\left(\frac{L}{4}+\frac{L}{4}\right) + \frac{L}{2\pi}\underbrace{\left(\sin\frac{\pi}{4} - \sin\frac{\pi}{4}\right)}_{=0}\right] = \frac{A^2}{2}\times\frac{L}{2} = \frac{A^2 L}{4}$$

$$\Rightarrow \boxed{A = \frac{2}{\sqrt{L}}}$$

b)

$$P = \int_0^{\frac{L}{8}} \psi^2 dx = \frac{4}{L}\int_0^{\frac{L}{8}} \cos^2\left(\frac{2\pi}{L}x\right) dx = \frac{4}{2L}\int_0^{\frac{L}{8}} \left[1 + \cos\left(\frac{4\pi}{L}x\right)\right] dx$$

$$= \frac{2}{L}\left[x\Big|_0^{\frac{L}{8}} + \frac{L}{4\pi}\sin\left(\frac{4\pi}{L}x\right)\Big|_0^{\frac{L}{8}} \right] = \frac{2}{L}\left[\frac{L}{8} + \frac{L}{4\pi}\sin\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{2}{L}\left[\frac{L}{8} + \frac{L}{4\pi}\right] = \frac{1}{4} + \frac{1}{2\pi} = \boxed{41\%}$$