Pb # 16.

a) For a receeding source $f_{obs} = \frac{\sqrt{1-v/c}}{\sqrt{1+v/c}} f_{source}$

$$f_{obs} = (1 - v/c)^{1/2} \left(1 + \frac{v}{c}\right)^{-1/2} f_{source}$$

For small x $(1-x)^n \simeq 1 - nx$

$$\Rightarrow f_{obs} \simeq \left(1 - \frac{v}{2c}\right)\left(1 - \frac{v}{2c}\right) f_{source}$$

$$\simeq \left(1 - \frac{v}{c} + \frac{v^2}{4c^2}\right) f_{source} \simeq \left(1 - \frac{v}{c}\right) f_{source}$$

(we neglected $\frac{v^2}{4c^2}$)

$$\frac{\Delta f}{f} = \frac{f_{obs} - f_{source}}{f_{source}} \simeq \frac{(1 - v/c) f_{source} - f_{source}}{f_{source}} \simeq -v/c$$

$$\Rightarrow \boxed{\frac{\Delta f}{f} \simeq -\frac{v}{c}}$$

b) We know that $c = \lambda f \Rightarrow f = \frac{c}{\lambda}$

differentiate $df = -\frac{c}{\lambda^2} d\lambda \Rightarrow \frac{df}{f} = -\frac{c}{\lambda^2} \frac{d\lambda}{f}$

$$\frac{df}{f} = -\frac{c}{\lambda^2} \frac{d\lambda}{c} \lambda = -\frac{d\lambda}{\lambda}$$

Change $d \to \Delta$ $\Rightarrow$ $\frac{\Delta\lambda}{\lambda} = -\frac{\Delta f}{f} \simeq \frac{v}{c}$

$$\Rightarrow \boxed{\frac{\Delta\lambda}{\lambda} \simeq \frac{v}{c}}$$