2. An electron is trapped in an infinitely deep potential well with width L=0.300 nm. The wave function for the electron is $\psi(x) = A \sin\left(\frac{n \pi x}{L}\right)$.

(a) Show that the normalization constant $A = \sqrt{\frac{2}{L}}$

(b) If the electron is in the ground state, what is the probability of finding the electron between x = 0.0 and 0.100 nm.

(c) Repeat the calculation of part (b) for the n = 3 state.

(d) Calculate the energy in electron-Volt for the electron in the n = 2 state.

a) $1 = \int_0^L |\psi|^2 dx = A^2 \int_0^L \sin^2 \frac{n\pi x}{L} = \frac{A^2}{2}\int_0^L \left(1 - \cos\frac{2n\pi x}{L}\right)dx$

$= \frac{A^2}{2}\left(x - \frac{L}{2n\pi}\sin\frac{2n\pi x}{L}\right)\Big|_0^L = \frac{A^2 L}{2} \Rightarrow \underline{A = \sqrt{\frac{2}{L}}}$

b) Given $L = 0.300\ nm$, $n = 1$, $0.100\ nm = \frac{L}{3}$

$P_{(0-0.100)nm} = \frac{2}{L}\int_0^{L/3} \sin^2\frac{\pi x}{L}\, dx = \frac{\cancel{2}}{\cancel{2}L}\int_0^{L/3}\left(1 - \cos\frac{2\pi x}{L}\right)dx$

$P_{0-0.1} = \frac{1}{L}\left(x - \frac{L}{2\pi}\sin\frac{2\pi x}{L}\right)\Big|_0^{L/3} = \frac{1}{L}\left(\frac{L}{3} - \frac{L}{2\pi}\sin\frac{2\pi}{3} - 0 + 0\right)$

$= \frac{1}{3} - \frac{1}{2\pi}\frac{\sqrt{3}}{2} = \underline{0.1955}$

c) for $n = 3$, $\psi = \frac{2}{L}\sin\frac{3\pi x}{L}$, $0.100 = \frac{L}{3}$

$P_{0-0.1} = \frac{2}{L}\int_0^{L/3}\sin^2\frac{3\pi x}{L}\, dx = \frac{\cancel{2}}{\cancel{2}L}\int_0^{L/3}\left(1 - \cos\frac{6\pi x}{L}\right) dx$

$= \frac{1}{L}\left(x - \frac{L}{6\pi}\sin\frac{6\pi x}{L}\right)\Big|_0^{L/3} = \frac{1}{L}\left(\frac{L}{3} - \frac{6}{6\pi}\sin 2\pi - 0 + 0\right)$

$= \frac{1}{3} = \underline{0.333}$

d) For $n = 2$ state, we have $2\frac{\lambda}{2} = L$, $\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{L}$

$K = \frac{p^2}{2m} = \frac{h^2}{2mL^2} = \frac{(6.62 \times 10^{-34}\ J\text{-}s)^2}{2 \times 9.11 \times 10^{-31} kg\ (0.300 \times 10^{-9} m)^2}$

$= 2.68 \times 10^{-18}\ J = \underline{16.7\ eV}$