

**Problem#6 (5 points)**

Certain surface waves in a fluid travel with a phase velocity  $\sqrt{\frac{b}{\lambda}}$ , where  $b$  is a constant.

(a) Do we have dispersion in this case? Explain.

Yes, because  $v_p$  depends on  $\lambda$ .

(b) Find the group velocity of a packet of surface waves in terms of the phase velocity.

$$v_g = v_p \Big|_{k_0} + k \frac{dv_p}{dk} \Big|_{k_0} \quad \text{where } k_0 \text{ is the central wavenumber.}$$

$$v_p = \sqrt{\frac{b}{\lambda}} = \sqrt{\frac{b}{2\pi k}} = \sqrt{\frac{b}{2\pi}} k^{-1/2}$$

$$\frac{dv_p}{dk} = \sqrt{\frac{b}{2\pi}} \cdot \frac{1}{2} k^{-3/2}$$

$$k \frac{dv_p}{dk} = \frac{1}{2} \sqrt{\frac{b}{2\pi}} k^{-1/2} = \frac{v_p}{2}$$

$$\Rightarrow v_g = v_p + \frac{v_p}{2} = \boxed{\frac{3v_p}{2} \Big|_{k_0}}$$