Ex 5.13

What are the limit of a classical oscillator having the Same total energy as the quantum oscillator in its ground state?

$$E = \frac{1}{2} k A^2 = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2}\hbar\omega$$

: amplitude of oscillations.

$$A = \sqrt{\frac{\hbar}{m\omega}} \qquad -A \le x \le A$$

The classical oscillator vibrates in the interval $-A \le x \le A$ having insufficient energy to exceed these limits.

Ex 5.14

Probability that a quantum oscillator in its ground state will be found outside the range permitted for a classical oscill. with the same energy.

$$P = \int_{-\infty}^{-A} |\Psi_0|^2 dx + \int_{A}^{\infty} |\Psi_0|^2 dx = 2\left(\frac{m\omega}{\pi\hbar}\right)^{1/2} \int_A^{\infty} e^{-\frac{m\omega}{\hbar}x^2} dx$$

set $x<z$ $\quad z = \sqrt{\frac{m\omega}{\hbar}}\, x$ and using $A = \sqrt{\frac{m\omega}{\hbar}}$ (corresponding to $z = 1$)

$$dx = \sqrt{\frac{\hbar}{m\omega}}\, dz$$

tabulated

$$P = \underbrace{\frac{2}{\sqrt{\pi}} \int_1^{\infty} e^{-z^2} dz} = \text{erf}(1) = 0.157 \simeq 16\%$$

The First excited state should be antisymmetric about the midpoint of the oscillator well $(x=0)$ with one node. This node must be at the origin so $\Psi_1(x) = x\, e^{-\alpha x^2}$ — substituting into S.E $\Rightarrow$ $E_1 = \frac{3}{2}\hbar\omega$ and so on.

In general $\boxed{E_n = \left(n+\frac{1}{2}\right)\hbar\omega}$ $\quad n = 0, 1, 2, \ldots$