The ground state is described by the wave function ground state energy

$$\psi_0(x) = C_0\, e^{-\frac{m\omega}{2\hbar}x^2} \quad \text{and} \quad E_0 = \frac{1}{2}\hbar\omega$$

this is a Gaussian form

Classically the particle oscillate between $-A$ and A

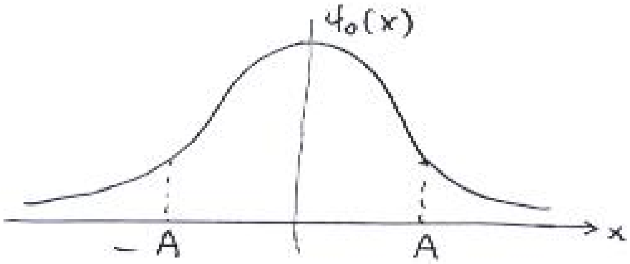
$E = \frac{1}{2}kA^2$ (A is the amplitude)

$A = \sqrt{\frac{\hbar}{m\omega}}$ ($\frac{1}{2}\omega^2 m A^2 = \frac{1}{2}\hbar\omega$

$A^2 = \frac{\hbar}{m\omega} \Rightarrow A = \sqrt{\frac{\hbar}{m\omega}}$

$x^2 = A^2 = \frac{\hbar}{m\omega}$

$$\psi_0(A) = C_0\, e^{-\frac{m\omega}{2\hbar}A^2} = C_0\, e^{-\frac{1}{2}}$$



Ex 5.12 Find C_0?

$$\psi_0(x) = C_0\, e^{-\frac{m\omega}{\hbar}x^2}$$

$$\int_{-\infty}^{+\infty} |\psi_0(x)|^2\, dx = 1 = C_0^2 \int_{-\infty}^{+\infty} e^{-\frac{m\omega}{\hbar}x^2}\, dx$$

$$\int_{-\infty}^{+\infty} e^{-ax^2}\, dx = \sqrt{\frac{\pi}{a}} \quad a > 0$$

in our case $a = \frac{m\omega}{\hbar} \Rightarrow$ $C_0^2 \sqrt{\frac{\pi\hbar}{m\omega}} = 1$

$$\Rightarrow C = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}$$