

The ground state is described by the wavefunction
 $\psi_0(x) = C_0 e^{-\frac{m\omega}{2\hbar}x^2}$ and ground state energy
 $E_0 = \frac{1}{2}\hbar\omega$
 this is a Gaussian form

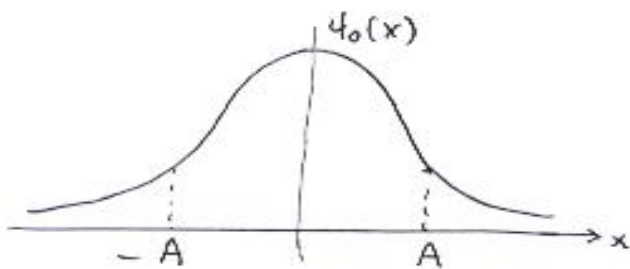
Classically the particle oscillates between $-A$ and A
 $E = \frac{1}{2}kA^2$ (A is the amplitude)

$$A = \sqrt{\frac{\hbar}{m\omega}} \quad \left(\frac{1}{2}m\omega^2 A^2 = \frac{1}{2}\hbar\omega \right)$$

$$A^2 = \frac{\hbar}{m\omega} \Rightarrow A = \sqrt{\frac{\hbar}{m\omega}}$$

$$x^2 = A^2 = \frac{\hbar}{m\omega}$$

$$\psi_0(A) = C_0 e^{-\frac{m\omega}{2\hbar}A^2} = C_0 e^{-\frac{1}{2}}$$



Ex 5.12 Find C_0 ?

$$\psi_0(x) = C_0 e^{-\frac{m\omega}{\hbar}x^2}$$

$$\int_{-\infty}^{+\infty} |\psi_0(x)|^2 dx = 1 = C_0^2 \int_{-\infty}^{+\infty} e^{-\frac{m\omega}{\hbar}x^2} dx$$

$$\int_{-\infty}^{+\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}} \quad a > 0$$

$$\text{in our case } a = \frac{m\omega}{\hbar} \Rightarrow C_0^2 \sqrt{\frac{\pi\hbar}{m\omega}} = 1$$

$$\Rightarrow C_0 = \left(\frac{m\omega}{\pi\hbar} \right)^{1/4}$$