$\Rightarrow$ Schrodinger equation becomes (time independent)

$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}\left(\frac{1}{2}m\omega^2 x^2 - E\right)\psi(x) \quad \text{— (1)}$$

It is beyond the scope of this course to solve this differential equation because of the term x^2 in the potential.

We can make a guess at the solution and verify by direct substitution.

The ψ_0 should "symmetric" about the midpoint of the pot. Well $x = 0$

ψ_0 should be "nodeless" but $\psi_0 \to 0$ when $|x| \to \infty$

Choose $\psi_0(x) = C_0\, e^{-\alpha x^2}$

C_0 and α are yet unknown.

$$\frac{d\psi_0}{dx} = -2\alpha x C_0\, e^{-\alpha x^2}$$

$$\frac{d^2\psi_0}{dx^2} = -2\alpha C_0\, e^{-\alpha x^2} + 4\alpha^2 x^2 C_0\, e^{-\alpha x^2}$$

$$= (4\alpha^2 x^2 - 2\alpha) C_0\, e^{-\alpha x^2} = (4\alpha^2 x^2 - 2\alpha)\psi_0(x) \quad -(2)$$

Compare (1) & (2)

$$4\alpha^2 = \frac{m^2\omega^2}{\hbar^2} \quad \text{or} \quad \boxed{\alpha = \frac{m\omega}{2\hbar}}$$

and $$2\alpha = \frac{2m}{\hbar^2}E = \frac{2m\omega}{2\hbar}$$

$$\boxed{E_0 = \frac{1}{2}\hbar\omega}$$