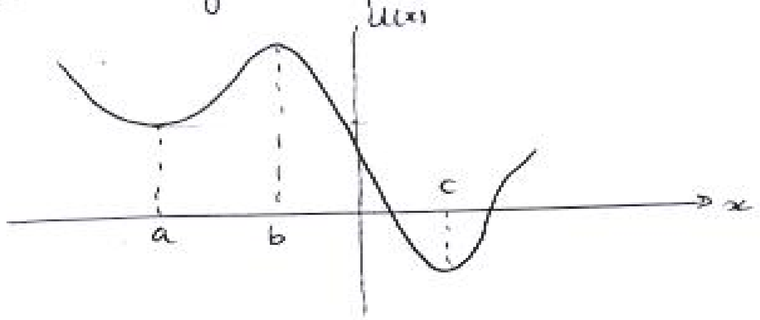
5.6 The Quantum Oscillator

A particle subject to a linear restoring force $F = -kx$

$\Rightarrow$ $U(x) = \frac{1}{2} k x^2$. (mass + spring) physical system but apply to any object having small oscillations about a point of stable equilibrium.

Consider the general potential function



a, b and c are equilibrium points because

$$F = -\frac{dU}{dx} = 0$$

a & c are <u>stable</u> equil. $\left(\frac{d^2U}{dx^2} > 0 \right)$

b is <u>unstable</u> equil. $\left(\frac{d^2U}{dx^2} < 0 \right)$

Near a or c $U(x)$ is a parabola

$$U(x) = U(a) + \frac{1}{2} K (x-a)^2$$

$k = \left. \frac{d^2U}{dx^2} \right|_a$ is the curvature of the parabola.

We can make $U(a) = 0$ (reference energy) at $\underline{a = 0}$.

$\Rightarrow$ $U(x) = \frac{1}{2} k (x \ \ )^2$ spring potential

The quantum oscillator is described by a potential

$$U(x) = \frac{1}{2} k x^2 = \frac{1}{2} m \omega^2 x^2$$