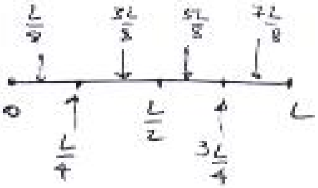
Ex 5.8 Probabilities for a Particle in a Box

find P $\quad \frac{L}{4} \le x \le \frac{3L}{4}$



$$P = \frac{2}{L}\int_{L/4}^{3L/4} \sin^2\left(\frac{n\pi}{L}x\right) dx$$

$$= \frac{2}{2L}\int_{L/4}^{3L/4} \left[1 - \cos\left(\frac{2\pi}{L}x\right)\right] dx$$

Ground state
$n=1$

$$= \frac{1}{L}\left[x\Big|_{L/4}^{3L/4} - \frac{L}{2\pi}\sin\left(\frac{2\pi}{L}x\right)\Big|_{L/4}^{3L/4}\right]$$

$$= \frac{1}{L}\left[\underbrace{\left(\frac{3L}{4} - \frac{L}{4}\right)}_{\frac{L}{2}} - \underbrace{\left(\frac{L}{2\pi}\right)(-1-1)}_{\frac{L}{\pi}}\right] = 0.818 \simeq 82\%$$

$$= \frac{1}{2} + \frac{1}{\pi} = 0.818$$

Exercice 3

A particle is in the n^{th} state

$$P = \frac{2}{L}\int_{L/4}^{3L/4} \sin^2\left(\frac{n\pi}{L}x\right) dx$$

$$= \frac{2}{2L}\int_{L/4}^{3L/4} \left[1 - \cos\left(\frac{2n\pi}{L}x\right)\right] dx$$

$$= \frac{1}{L}\left\{\left(\frac{3L}{4} - \frac{L}{4}\right) - \frac{L}{2n\pi}\left[\sin\left(\frac{2n\pi}{L}\cdot\frac{3L}{4}\right) - \sin\left(\frac{2n\pi}{L}\cdot\frac{L}{4}\right)\right]\right\}$$

$$= \frac{1}{L}\left\{\frac{L}{2} - \frac{L}{2n\pi}\left(\sin\frac{6n\pi}{4} - \sin\frac{2n\pi}{4}\right)\right\}$$

$n \to \infty$ the second term $\to 0$ $\Rightarrow$ $\boxed{P = \frac{1}{2}}$