Exercice 2 :

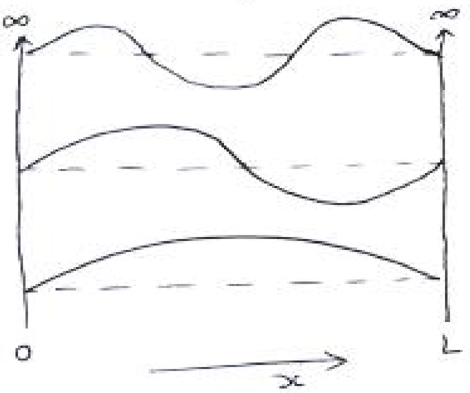
$$E_1 = \frac{1}{2} m_e v^2 = \frac{\pi^2 \hbar^2}{2 m_e L^2} \Rightarrow v = \sqrt{\frac{\pi^2 \hbar^2}{m_e L^2}}$$

$$\hbar = \frac{h}{2\pi} \qquad v = \sqrt{\frac{h}{4 m_e L^2}} =$$

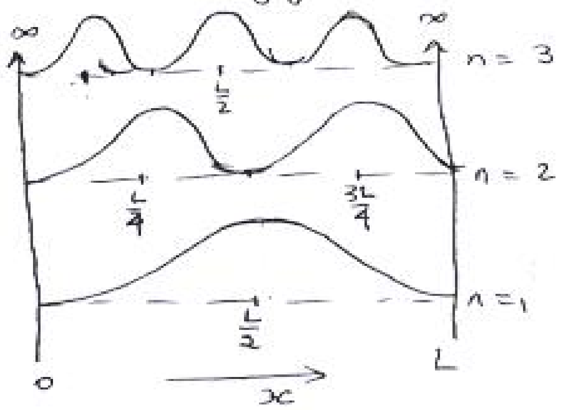
Return to the wave function

$$k = \frac{n\pi}{L} \quad \text{and} \quad B = 0 \Rightarrow \boxed{\psi_n(x) = A \sin\left(\frac{n\pi}{L} x\right)} \qquad 0 \leq x \leq L, \; n = 1, 2, \ldots$$

For each value of the quantum number n there is a wave function $\psi_n(x)$ describing the state of the particle with energy E_n.



$\psi_n(x)$



$|\psi_n(x)|^2$

r $n = 1$ P is largest at $x = \frac{L}{2}$ – this is the most probable position.

r $n = 2$ P is maximum at $x = \frac{L}{4}$ and $\frac{3L}{4}$

r $n = 3$ P is ~~maximum~~ minimum at $x = \frac{L}{3}$, $\frac{2L}{3}$, ... It is impossible to find the particle at those locations.

Let us normalize the wave function ψ_n

$$1 = \int_{-\infty}^{+\infty} |\psi_n(x)|^2 \, dx = A^2 \int_0^L \sin^2\left(\frac{n\pi}{L} x\right) dx$$

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$$1 = \frac{A^2}{2} \int_0^L \left(1 - \cos\left(\frac{2\pi n x}{L}\right)\right) dx = A^2 \frac{L}{2} \Rightarrow A = \sqrt{\frac{2}{L}}$$