$\psi(0) = 0 \qquad x = L \qquad \Rightarrow \qquad A \sin kL = 0$

$\Rightarrow \qquad kL = n\pi \qquad n = 1, 2, 3, \ldots$

$\left[k = \frac{2\pi}{\lambda} \quad \Rightarrow \quad \frac{\lambda}{2\pi} L = n\pi \quad \Rightarrow \quad \frac{\lambda}{2} = \frac{n\pi^2}{L} \right]$

$k = \frac{n\pi}{L} = \frac{\sqrt{2mE}}{\hbar} \quad \Rightarrow \quad \boxed{E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}} \quad n = 1, 2, 3, \ldots$

n is the quantum number called. energy is quantized

gn ⇒

$9E_1$ The ground state corresponds to $n = 1$

$4E_1 \qquad E_1 = \frac{\pi^2 \hbar^2}{2mL^2}$ or zero-point energy > 0

E_1 and $E_n = n^2 E_1 \quad n = 2, 3, 4, \ldots$ are the excited states. [Do not think that the particle jump from time t to another]

$E = 0$ is not allowed, the particle can never be at rest.

$n = 0 \qquad E = 0 \qquad \frac{d^2\psi}{dx^2} = 0 \Rightarrow \psi = ax + b$

$\psi(0) = 0 \Rightarrow b = 0 \qquad \psi(L) = 0 \Rightarrow a = 0 \Rightarrow \psi_{(x)} = 0$ everywhere. $|\psi|^2 = 0$ the particle cannot be found ⇒ no description is possible when $E = 0$. Classically $E = 0$ is allowed and E is continuous.

Ex 5.6 Energy Quantization for Macroscopic Objects.

$m = 1.00$ mg $\qquad L = 1.00$ cm

a) $E_1 = \frac{\pi^2 \hbar^2}{2mL^2} = \frac{1}{2} m v^2 \Rightarrow v = \sqrt{\frac{2\pi^2\hbar^2}{2m^2L^2}} = \sqrt{\frac{h^2}{4mL^2}}$

$\hbar = \frac{h}{2\pi} \qquad v = \sqrt{\frac{(6.63 \times 10^{-34})^2}{4(1 \times 10^{-3})(0.01)^2}} = 3.31 \times 10^{-26}$ m/s

This is too small. The object can be considered to be at rest.