5.7 Expectation Values

Quantities such as E (energy of the particle) are called "sharp". It can be measured exactly. ~~and will always have the same value~~ for

Quantities such as x (position of the particle) are called "fuzzy" for which the wavefunction Ψ furnishes only probabilities.

In classical mechanics we can write the average particle position from any data set as:

$$\overline{x} = \sum x\, P(x)$$

(x: various x values; $P(x)$: probability of occurence.)

In quantum mechanics $P(x) = |\Psi|^2$ and the average value of x written $\langle x \rangle$ is called the "expectation value". Then

$$\boxed{\langle x \rangle = \int_{-\infty}^{+\infty} x\, |\Psi(x,t)|^2\, dx}$$

average position of a particle.

Any function of x, say $f(x)$

$$\langle f \rangle = \int_{-\infty}^{+\infty} f(x)\, |\Psi(x,t)|^2\, dx .$$

If $f(x) = U(x)$ (potential Energy)

$$\langle U \rangle = \int_{-\infty}^{+\infty} U(x)\, |\Psi(x,t)|^2\, dx$$

If $f(x) = x^2$ the quantum uncertainty in particle position may be found.

$$\boxed{\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}}$$

The position is sharp only if $\Delta x = 0$

(classically it is the standard deviation = amount of scatter about the average value)