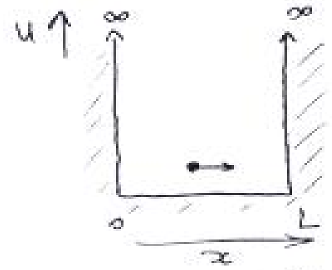
5.4 The Particle in a Box

As a simple application of quantum mechanics, let us consider a single particle whose motion is restricted by the reflecting walls of a one-dim infinite well (or barrier) as shown in Fig. 5.8c. and given by

$$U(x) = 0 \qquad 0 \le x \le L$$
$$U(x) = \infty \qquad x < 0 \text{ and } x > L.$$



Classically the particle simply bounces back and forth between the walls. Its speed remains constant, as does its K.E. There are no restrictions on p and E.
The Quantum description is different and lead to the interesting phenomena of energy quantization.
Outside the walls the particle can never be there $\Rightarrow \psi(x) = 0$
Inside the box $U(x) = 0 \Rightarrow$

$$\frac{d^2\psi(x)}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \quad \text{or} \quad \frac{d^2\psi(x)}{dx^2} + k^2\psi = 0$$

Where $k = \sqrt{\frac{2mE}{\hbar^2}} = \frac{\sqrt{2mE}}{\hbar}$

The general solution is $\psi(x) = A \sin kx + B \cos kx$

where A & B are constants evaluated using boundary Conditions. $\psi(x) = 0$ at $x = 0$ and $x = L$

$$\psi(0) = B = 0 \Rightarrow B = 0$$
$$\Rightarrow \psi(x) = A \sin kx$$