b # 27.

$$P = \int_0^{4a_0} P_{2s}(r)\, dr = \int_0^{4a_0} r^2 |R_{2s}(r)|^2 dr$$

$$= \int_0^{4a_0} r^2 \left(\frac{1}{2a_0}\right)^3 \left(2 - \frac{r}{a_0}\right)^2 e^{-\frac{r}{a_0}}\, dr$$

$$= \frac{1}{8a_0} \int_0^{4a_0} \left(\frac{r}{a_0}\right)^2 \left(2 - \frac{r}{a_0}\right)^2 e^{-\frac{r}{a_0}}\, dr$$

let $z = \frac{r}{a_0} \Rightarrow r = z a_0 \quad dr = a_0\, dz$

$$P = \frac{\cancel{a_0}}{8\cancel{a_0}} \int_0^4 z^2 (4 + z^2 - 4z)\, e^{-z} dz$$

$$= \frac{1}{8} \int_0^4 (4z^2 - 4z^3 + z^4)\, e^{-z} dz$$

$$= \frac{1}{8}\left[4\int_0^4 z^2 e^{-z} dz - 4\int_0^4 z^3 e^{-z} dz + \int_0^4 z^4 e^{-z} dz \right]$$

b # 29.

$\langle r \rangle = 1.5\, a_0$

$\Delta r = \sqrt{\langle r^2 \rangle - \langle r \rangle^2}$

$\langle r \rangle^2 = 2.25\, a_0^2$

$$\langle r^2 \rangle = \int_0^\infty r^2\, r^2 |R_{1s}(r)|^2 dr = \frac{4}{a_0^3} \int_0^\infty r^4 e^{-\frac{2r}{a_0}} dr$$

$\frac{2r}{a_0} = z \quad r = z\,\frac{a_0}{2} \quad dr = \frac{a_0}{2} dz$

$$\langle r^2 \rangle = \frac{4}{a_0^3} \left(\frac{a_0}{2}\right)^5 \int_0^\infty z^4 \underbrace{e^{-z} dz}_{4!} = 3\, a_0^2$$