

Three vectors are given in unit vector notation as

$$\vec{A} = 2\hat{i} + 4\hat{j} - 4\hat{k}, \quad \vec{B} = -3\hat{i} + 2\hat{j} - 4\hat{k}, \quad \text{and} \quad \vec{C} = 6\hat{j} + 10\hat{k}$$

(a) Find the vector \vec{D} such that $\vec{D} = 3\vec{A} - \frac{1}{2}\vec{C}$

$$\begin{aligned} \vec{D} &= 3(2\hat{i} + 4\hat{j} - 4\hat{k}) - \frac{1}{2}(6\hat{j} + 10\hat{k}) \\ &= (6\hat{i} + 12\hat{j} - 12\hat{k}) - (3\hat{j} + 5\hat{k}) \\ &= 6\hat{i} + 9\hat{j} - 17\hat{k} \end{aligned}$$

(b) Find $\vec{A} \cdot \vec{B}$

$$\begin{aligned} \vec{A} \cdot \vec{B} &= (2\hat{i} + 4\hat{j} - 4\hat{k}) \cdot (-3\hat{i} + 2\hat{j} - 4\hat{k}) \\ &= -6 + 8 + 16 = 18 \end{aligned}$$

(c) Find $\vec{A} \times \vec{B}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & -4 \\ -3 & 2 & -4 \end{vmatrix}$$

$$\begin{aligned} &= (-16 + 8)\hat{i} - (-8 - 12)\hat{j} + (4 + 12)\hat{k} \\ &= -8\hat{i} + 20\hat{j} + 16\hat{k} \end{aligned}$$