PHYS 301 – Term 151

Quiz#2-chapter 3

11/10/2015

Name: Key Id#:

Consider a simple harmonic oscillator. Calculate the space averages of the kinetic and potential energies. Are they equal? Discuss the results.

Consider x(t) = A cos(wt) where A is the amplitude of the motion and w is the angular frequency.

$$x(t) = A\cos\omega t \qquad \dot{x}(t) = -A\omega\sin\omega t$$

$$\langle K\rangle = \frac{1}{2}m\,\frac{1}{A}\int_0^A \dot{x}^2\,dx = \frac{1}{2}m\,\frac{1}{A}\int_0^A A^2\omega^2\sin^2\omega t\,dx$$

$$= \frac{1}{2}m\,\frac{1}{A}\int_0^A A^2\omega^2(1-\cos^2\omega t)\,dx$$

$$= \frac{1}{2}m\,\frac{1}{A}\left[\int_0^A A^2\omega^2\,dx - \int_0^A A^2\omega^2\cos^2\omega t\,dx\right.$$

$$= \frac{1}{2}m\,\frac{1}{A}\left[A^2\omega^2\int_0^A dx - \omega^2\int_0^A x^2\,dx\right]$$

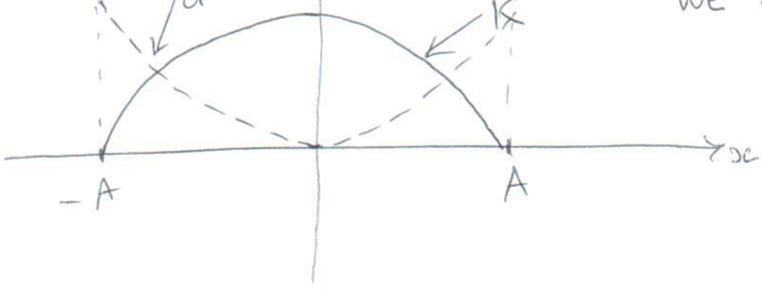
$$= \frac{1}{2}m\,\frac{1}{A}\left[A^2\omega^2 A - \omega^2\frac{A^3}{3}\right] = \frac{1}{2}m\frac{1}{A}\left(\frac{2A^3\omega^2}{3}\right)$$

$$\boxed{\langle K\rangle = \frac{m\omega^2A^2}{3}}$$

$$\langle U\rangle = \frac{1}{2}k\,\frac{1}{A}\int_0^A x^2\,dx = \frac{1}{2}k\,\frac{1}{A}\,\frac{A^3}{3} = \frac{kA^2}{6}$$

$$k = m\omega^2 \Rightarrow \boxed{\langle U\rangle = \frac{m\omega^2A^2}{6}}$$

We see that $\langle K\rangle = 2\langle U\rangle$



We see that the area under the cure for K is always greater than the area under the curve for U!