PHYS 301 – Term 151
Quiz#4-chapter 6
05/11/2015

Name: Key Id#:

Show that the geodesic on the surface of a right cylinder is a segment of a helix.

$$ds = \sqrt{dr^2 + r^2 d\phi^2 + dz^2} \qquad r = a \Rightarrow dr = 0$$

$$S = \int ds = \int \sqrt{a^2 d\phi^2 + dz^2}$$

$$= \int \underbrace{\sqrt{a^2 + \left(\frac{dz}{d\phi}\right)^2}}_{f(\dot{z})} \, d\phi = \int \sqrt{a^2 + \dot{z}^2} \, d\phi \qquad \dot{z} = \frac{dz}{d\phi}$$

Euler's equation $\frac{\partial f}{\partial z} - \frac{d}{d\phi}\left(\frac{\partial f}{\partial \dot{z}}\right) = 0$

$$\frac{\partial f}{\partial z} = 0 \qquad \frac{\partial f}{\partial \dot{z}} = \frac{1}{\cancel{2}} \frac{\cancel{2}\dot{z}}{\sqrt{a^2+\dot{z}^2}} = \frac{\dot{z}}{\sqrt{a^2+\dot{z}^2}}$$

$$\frac{d}{d\phi}\left(\frac{\partial f}{\partial \dot{z}}\right) = 0 \Rightarrow \frac{\partial f}{\partial \dot{z}} = \text{Constant} = C_1$$

$$\frac{\dot{z}}{\sqrt{a^2+\dot{z}^2}} = C_1 \Rightarrow \dot{z}^2 = a^2 C_1^2 + \dot{z}^2 C_1^2$$

$$\dot{z}^2(1 - C_1^2) = a^2 C_1^2 \Rightarrow \dot{z} = \sqrt{\frac{a^2 C_1^2}{1 - C_1^2}} = \text{Constant} = K$$

$$\frac{dz}{d\phi} = K \Rightarrow z = K\phi + C_2 \qquad \phi = 0 \text{ when } z = 0 \Rightarrow C_2 = 0$$

$$\left.\begin{aligned} z &= K\phi \\ x &= a\cos\phi \\ y &= a\sin\phi \end{aligned}\right\} \text{equation of a helix !}$$