King Fahd University of Petroleum & Minerals
Department of Physics
Physics 301
Term 051
Quiz #3

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Name: Key Id#:

The differential equation for a damped harmonic oscillator is

$$\ddot{x} + 2\beta\dot{x} + \omega_o^2 x = 0$$

Given that m = 0.05 kg and k = 5.0 N/m, discuss the nature of the oscillations, sketch the graph of x vs. t and give the general solution for the initial conditions x (0) = 0.1 m and $\dot{x}(0) = 0$, for the following two cases:

(a) $\beta = 10$ rad/s.
(b) $\beta = 1$ rad/s.

a) $\omega_o = \sqrt{\frac{k}{m}} = \sqrt{100} = 10$ rad/s

$\omega_o^2 = \beta^2 \Rightarrow$ critical damping

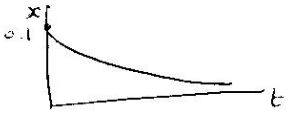
$x(t) = (A + Bt)\, e^{-\beta t}$

$x(0) = A = 0.1$ m

$\dot{x}(t) = B\, e^{-\beta t} - \beta (A + Bt)\, e^{-\beta t}$

$\dot{x}(0) = B - \beta A = 0 \Rightarrow B = \beta A = 10 \times 0.1 = 1$ m/s

$$\boxed{x(t) = (0.1 + t)\, e^{-10t}}$$



b) $\beta^2 < \omega_o^2 \Rightarrow$ underdamping

$x(t) = A\, e^{-\beta t} \cos(\omega_1 t - \delta)$

$x(0) = A \cos\delta = 0.1$

$\dot{x}(t) = -\beta A\, e^{-\beta t} \cos(\omega_1 t - \delta) - A\omega_1\, e^{-\beta t} \sin(\omega_1 t - \delta)$

$\dot{x}(0) = -\beta A \cos\delta + A\omega_1 \sin\delta = 0 \Rightarrow \tan\delta = \frac{\beta}{\omega_1} = \frac{1}{\sqrt{99}}$

$\Rightarrow \delta = 0.1$ rad

$0.1 = A \cos(0.1) \Rightarrow A = 0.1$ m

$$\boxed{x(t) = 0.1\, e^{-t} \cos\left(\sqrt{99}\, t - 0.1\right)}$$