King Fahd University of Petroleum & Minerals
Department of Physics
Physics 301 - Term 051
Quiz #2

Dr. A. Mekki

Name: Key Id#:

A boat with initial velocity v_0 is launched on a lake. The boat is slowed by the water by a force $F = -\alpha e^{+\beta v}$, where α and β are constants and v is the speed of the boat.

(a) Find an expression for the speed of the boat as a function of time and the other constants.
(b) Find the time the boat will come to rest.
(c) Find an expression for the displacement as a function of time.

Given: $\int \ln(ax+b)dx = \frac{ax+b}{a}\ln(ax+b) + x$

a) $F = ma \Rightarrow m\frac{dv}{dt} = -\alpha e^{\beta v}$

$$\frac{dv}{e^{\beta v}} = -\frac{\alpha}{m}dt \Rightarrow \int_{v_0}^{v} e^{-\beta v} dv = -\frac{\alpha}{m}\int_0^t dt$$

$$\Rightarrow -\frac{1}{\beta} e^{-\beta v}\Big|_{v_0}^{v} = -\frac{\alpha}{m} t$$

$$+\frac{1}{\beta}\left(e^{-\beta v} - e^{-\beta v_0}\right) = \frac{\alpha}{m} t$$

$$\Rightarrow e^{-\beta v} = \frac{\beta\alpha}{m} t + e^{-\beta v_0}$$

$$\Rightarrow \boxed{v(t) = -\frac{1}{\beta}\ln\left[\frac{\alpha\beta}{m}t + e^{-\beta v_0}\right]}$$

b) The boat comes to rest $\Rightarrow v_f = 0$

$$\Rightarrow 0 = -\frac{1}{\beta}\ln\left[\frac{\alpha\beta t_f}{m} + e^{-\beta v_0}\right] \Rightarrow \frac{\alpha\beta}{m}t_f + e^{-\beta v_0} = 1$$

$$\Rightarrow \boxed{t_f = \frac{m}{\alpha\beta}\left(1 - e^{-\beta v_0}\right)}$$

c) $v = \frac{dx}{dt} \Rightarrow x = \int_0^x v\,dt = -\frac{1}{\beta}\int_0^x \ln\left[\frac{\alpha\beta}{m}t + e^{-\beta v_0}\right]dt$

$$\Rightarrow x + C = -\frac{1}{\beta}\left[\frac{\frac{\alpha\beta}{m}t + e^{-\beta v_0}}{\frac{\alpha\beta}{m}}\ln\left(\frac{\alpha\beta}{m}t + e^{-\beta v_0}\right)\Big|_0^t - x\Big|_0^t\right]$$

$t=0 \quad x=0 \Rightarrow C = -\frac{1}{\beta}\frac{e^{-\beta v_0}}{\frac{\alpha\beta}{m}}\ln e^{-\beta v_0} = -\frac{1}{\beta}\frac{v_0 m}{\alpha\beta}e^{-\beta v_0}$

$$\Rightarrow \boxed{X_{(t)} = -\frac{m v_0}{\alpha\beta} e^{-\beta v_0} + \frac{t}{\beta} - \frac{\alpha m}{\alpha\beta^2}\left[\frac{\alpha\beta}{m}t + e^{-\beta v_0}\right]\ln\left(\frac{\alpha\beta t}{m} + e^{-\beta v_0}\right)}$$