# King Fahd University of Petroleum & Minerals
## Department of Physics
### Physics 301 - Term 051
### Quiz #1

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5 **Q.1**

Consider the following rotation matrix:

$$\lambda = \begin{pmatrix} \sqrt{3}/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & \sqrt{3}/2 \end{pmatrix}$$

(a) Prove that it is orthogonal.
(b) Find $\lambda^{-1}$.
(c) Find the point with coordinates $(x_1', x_2', x_3')$ obtained by transforming the point with coordinates $(1/2, 0, \sqrt{3}/2)$ under $\lambda$.
(d) What is the angle of rotation and axis of rotation.

3 **Q.2**

(a) Write the infinitesimal displacement vector $d\vec{s}$ in cylindrical coordinates.
(b) Find the velocity in cylindrical coordinates.
(c) Find $v^2$ in cylindrical coordinates.

Choose one of these two questions:

2 **Q.3**

Show that $\vec{\nabla}(\ln|\vec{r}|) = \dfrac{\vec{r}}{r^2}$

(Hint: $|\vec{r}| = \sqrt{x_1^{\ 2} + x_2^{\ 2} + x_3^{\ 2}} = \left(\sum_i x_i^{\ 2}\right)^{1/2}$, $\dfrac{d}{dx} Ln(x) = \dfrac{dx}{x}$ )

2 **Q.4**

Show that $\vec{A}(\vec{B}\times\vec{C}) = \vec{C}(\vec{A}\times\vec{B})$

# Phys 301 - Quiz #1 - Solution
## Term 051

**Q1.**

a) $\lambda$ is orthogonal if $\lambda \lambda^t = \mathbb{1}$

$$\begin{pmatrix} \sqrt{3}/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & \sqrt{3}/2 \end{pmatrix} \begin{pmatrix} \sqrt{3}/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & \sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} \frac{3}{4}+\frac{1}{4} & 0 & +\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4} & 0 & \frac{1}{4}+\frac{3}{4} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \mathbb{1}$$

$\uparrow$ identity matrix!

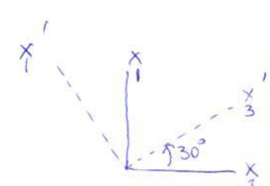
$\Rightarrow$ $\lambda$ is an orthogonal matrix

b) Since $\lambda$ is orthogonal, $\lambda^{-1} = \lambda^t \Rightarrow \lambda^{-1} = \begin{pmatrix} \sqrt{3}/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & \sqrt{3}/2 \end{pmatrix}$

c) $$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & \sqrt{3}/2 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ \sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/4 - \sqrt{3}/4 \\ 0 \\ 1/4 + \frac{3}{4} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

d) $\cos\theta = \sqrt{3}/2 \Rightarrow \theta = 30^\circ = \frac{\pi}{6}$

The axis of rotation is $x_2$




$x_1' = \cos 30^\circ \, \hat{e}_1 - \sin 30^\circ \, \hat{e}_3$

$x_2' = \sin 30^\circ \, \hat{e}_1 + \cos 30^\circ \, \hat{e}_3$

$$\Rightarrow \lambda = \begin{pmatrix} \cos 30^\circ & 0 & -\sin 30^\circ \\ 0 & 1 & 0 \\ \sin 30^\circ & 0 & \cos 30^\circ \end{pmatrix}$$

Q2. In cylindrical coordinates

$$d\vec{s} = dr\,\hat{e}_r + r\,d\theta\,\hat{e}_\theta + dz\,\hat{e}_z$$ (see figure F-2)

$$\vec{v} = \frac{d\vec{s}}{dt} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta + \dot{z}\,\hat{e}_z$$

$$v^2 = \vec{v}\cdot\vec{v} = \dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2$$ (we have used $\hat{e}_i \cdot \hat{e}_j = \delta_{ij}$)

Q3.

$$\vec{\nabla}\,\mathrm{Ln}\,(|\vec{r}|) = \sum_i \frac{\partial}{\partial x_i} \mathrm{Ln}\left(\sum_j x_j^2\right)^{1/2} \hat{e}_i$$

$$= \sum_i \frac{1}{2}\, 2x_i \,\frac{\left(\sum_j x_j^2\right)^{-1/2}}{\left(\sum_j x_j^2\right)^{1/2}}\,\hat{e}_i$$

$$= \sum_i x_i \frac{1}{\underbrace{\sum_j x_j^2}_{|\vec{r}|^2}}\,\hat{e}_i = \frac{1}{r^2}\underbrace{\sum_i x_i\,\hat{e}_i}_{\vec{r}}$$

$$= \frac{\vec{r}}{r^2}$$

$$\Rightarrow \vec{\nabla}\,\mathrm{Ln}(|\vec{r}|) = \frac{\vec{r}}{r^2}$$

Q4.

$$(\vec{B}\times\vec{C})_i = \sum_{jk} \varepsilon_{ijk} B_j C_k$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \sum_{ijk} \varepsilon_{ijk} A_i B_j C_k$$

$$(\vec{A}\times\vec{B})_i = \sum_{jk} \varepsilon_{ijk} A_j B_k$$

$$\vec{C}\cdot(\vec{A}\times\vec{B}) = \sum_{ijk} \varepsilon_{ijk} C_i A_j B_k = \sum_{ijk} \varepsilon_{jki} A_j B_k C_i$$

$$= \sum_{ijk} \varepsilon_{ijk} A_i B_j C_k = \vec{A}\cdot(\vec{B}\times\vec{C})$$

$$\Rightarrow \vec{C}\cdot(\vec{A}\times\vec{B}) = \vec{A}\cdot(\vec{B}\times\vec{C})$$