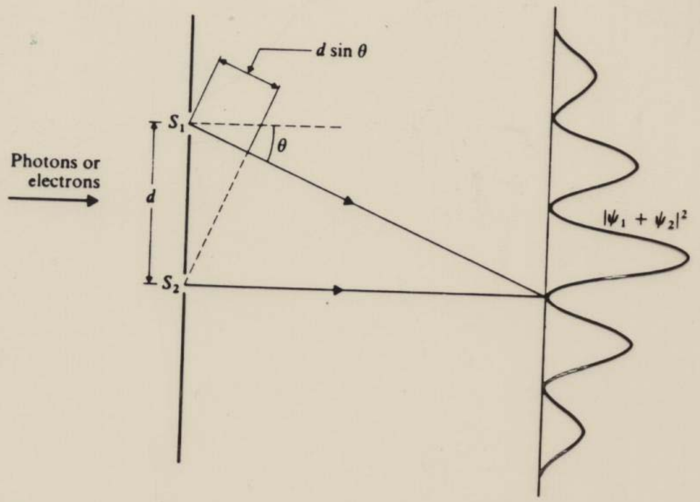


Fig. 3.3 Double-slit diffraction pattern of particles or photons. The first minimum occurs at an angle such that

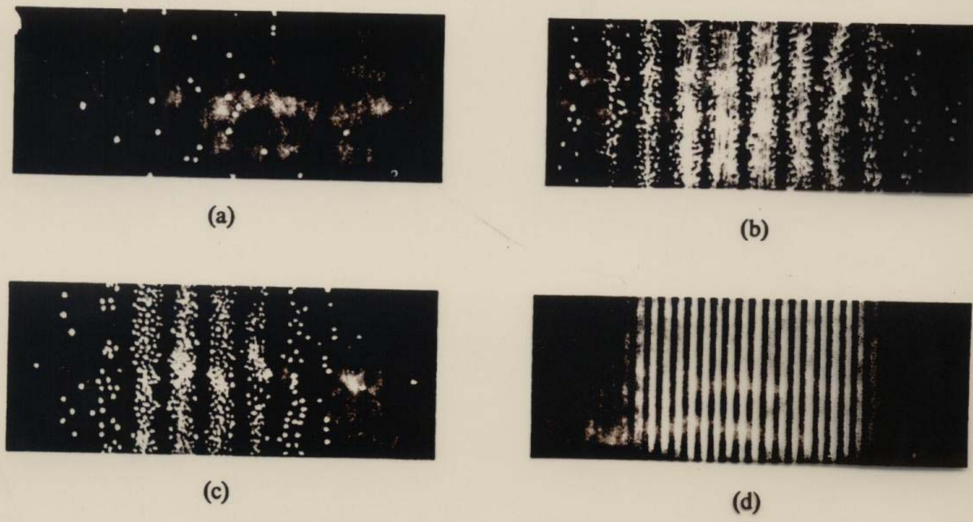


Fig. 3.4 Growth of a double-slit pattern on a photographic film when a beam of electrons or photons is incident on a double-slit system. (a) The pattern obtained when the film is struck by 28 electrons, (b) 1000 electrons, (c) about 10,000 electrons. (The dots are much bigger than actual size.) Note that there are no dots in the region of interference minima. (From Elisha R. Huggins, *Physics 1*, New York: W. A. Benjamin, 1968, page 510) (d) Double-slit pattern obtained when the film is exposed to millions of electrons or photons.

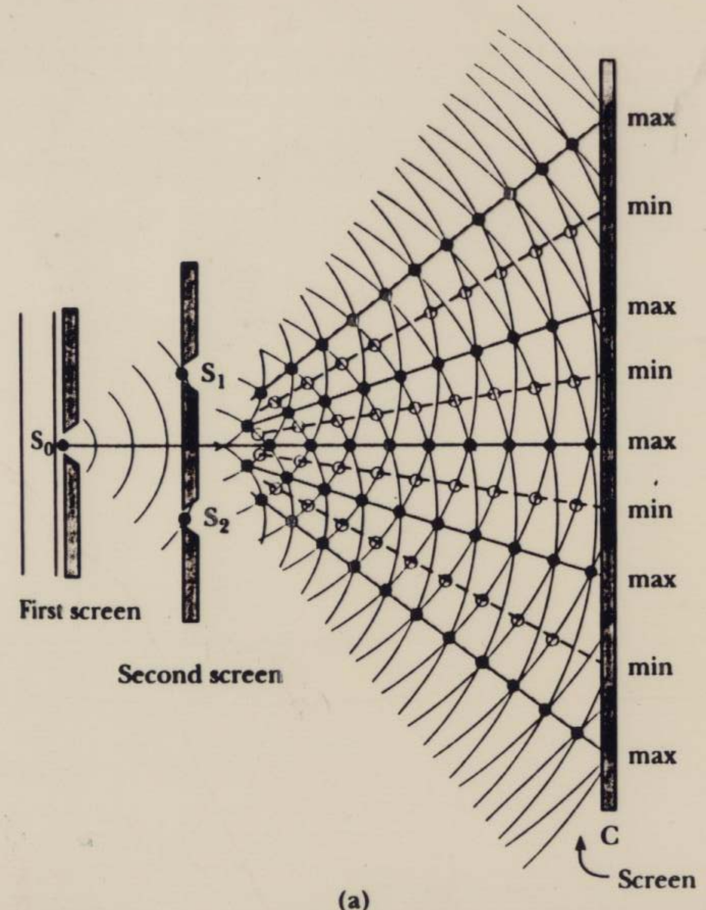


FIGURE 37.1 (a) Schematic diagram of Young's double-slit experiment. The narrow slits act as wave sources. Slits S_1 and S_2 behave as coherent sources that produce an interference pattern on screen C. (Note that this drawing is not to scale.) (b) The fringe pattern formed on screen C could look like this.

It is to be noted that while ψ has no physical meaning $|\psi|^2$ has. The branch of physics which deals with the problems of finding ψ and thus describing the behavior of wave packets (and hence the motion of particles) is called "quantum mechanics". This exploration was done independently by Schrödinger/ who used wave mechanics (1926) and by Born, Heisenberg and Jordan (1925) who used matrices to discuss the behavior of matter waves.
It was later found that both techniques gave the same results.

4.7 The Wave - Particle Duality

* The Description of Electrons Diffraction in Terms of ψ

We have seen evidence for both the wave properties and the particle properties of electrons. Well, is the electron a wave or a particle?

"Electrons behaves either as particles or waves depending on the kind of experiments performed on them."

Bohr complementarity principle: Both wave and particle views are needed and they complement each other to fully describe the electron.

A Good example to see the wave - particle duality is to consider a double - slit electron diffraction experiment.

4.6 If Electrons are Waves, what's Waving?

A de Broglie matter wave is represented by a wave function $\psi(x, y, z, t)$. [The quantity $\psi^* \psi = |\psi|^2$ represents the probability per unit volume of finding the particle at a time t in a small volume of space centered on (x, y, z)] no need

According to Max Born (1926)

The wavefunction $\psi(x, y, z, t)$ is a quantity such that the product $|\psi(x,y,z,t)|^2$ ~~dv~~ is the probability of observing a particle in a volume element $dv = dx\,dy\,dz$ at a time t.

If the system is stationary, i.e., independent of time

$$|\psi(x,y,z)|^2 dv = \psi^*(x,y,z)\, \psi(x,y,z)\, dv$$

ψ^* is the complex conjugate of ψ.

If $\psi = a + ib \quad \psi^* = a - ib$

$$\psi = A e^{i\theta} \qquad \psi^* = A e^{-i\theta}$$

$$\left\{ \begin{array}{l} \text{The probability of observing a particle} \\ \text{between } x \text{ and } x + dx \text{ is} \end{array} \right\} \propto |\psi(x)|^2 dx$$

wherever ψ is large, the probability of finding the particle is large and vice versa.

ψ ~~...~~ is not a measurable disturbance like in the case of water or sound waves.

ψ is used to calculate the probability of finding a particle at time t in a volume $dv = dx\,dy\,dz$

$$df' \geq \frac{1}{4\pi\tau}$$ (10)

$$\Delta f = f - f'$$

$$\Rightarrow \; dv = \frac{c}{f}\frac{1}{4\pi\tau} = \frac{c}{\pi f \tau}$$

$$f' = f - \Delta f$$

$$\Delta p_x = m\,\Delta v = \frac{mc}{4\pi f \tau}$$

$$\Delta p = \frac{(1.67\times10^{-27})(3\times10^{8})}{4\pi\,(6\times10^{14})(10^{-8})} = 6.65\times10^{-27}\ kg.\frac{m}{s}$$

b) Estimate $\Delta x = ?$ $\quad \Delta x = \Delta v\,\Delta t = \Delta v\,\tau$

Change in momentum of the atom $= \frac{hf'}{c} = m\,\Delta v$

$$\Delta v = \frac{hf'}{mc}$$

The precise time of emission of the photon is uncertain, this introduces an uncertainty in the atom's position.

$$\Delta x = (\Delta v)\,\tau = \frac{hf'\tau}{mc} = \frac{(6\times10^{14})(1\times10^{-8})(6.63\times10^{-34})}{(1.67\times10^{-27})(3\times10^{8})}$$

$$= 8.0\times10^{-9}\,m$$

c) $\Delta x\ \Delta p_x = ?$

$$8\times10^{-9}\times 6.65\times10^{-27} = 5.28\times10^{-35}.$$

$$\frac{h}{2\pi} = \quad \frac{\hbar}{2} = 5.29\times10^{-35}$$

$$\Rightarrow \quad \Delta x\ \Delta p_x = \frac{\hbar}{2}$$

Ex 4.13 Hydrogen atom

Measuring the Momentum of an Atom from the Doppler Shift

$\tau = \Delta t = 10^{-8}$ sec

at rest $\lambda = 500\,\text{nm}$

Doppler shift in freq. when the atom is moving is measured to be $\Delta f = 1 \times 10^{10}$ Hz

find p and Δp for the atom

$v \ll c \qquad \dfrac{v}{c} = \dfrac{\Delta f}{f} = \dfrac{f' - f}{f}$

f : freq. radiated / when atom at rest

f' : " " " " is moving with speed v

$$\frac{v}{c} = \frac{\Delta f}{f} = \frac{\Delta f}{c}\lambda = 5 \times 10^{3}\ \text{m/s}$$

$$p = mv = (1.67 \times 10^{-27})(3 \times 10^{3}) = 8.35 \times 10^{-24}\ \text{Kg}\cdot\frac{\text{m}}{\text{s}}$$

There is uncertainty in v because measurement of f' is uncertain because the time τ determined the uncertainty in the energy of the excited state and hence in the freq. of the transition.

now $\quad \dfrac{v}{c} = \dfrac{f'}{f} - 1$

$$dv = \frac{c}{f}\, df'$$

using $\Delta E\, \Delta t \geq \dfrac{\hbar}{2}$ and $\Delta E = h\, \Delta f'$

and $\Delta t = \tau \quad \Rightarrow \Delta f' = \dfrac{1}{4\pi\tau}$

b) $\lambda = 500\ nm$

$\frac{\Delta f}{f_0} = ?$ $\Leftarrow$ fractional broadning

$$f_0 = \frac{c}{\lambda} \Rightarrow \frac{\Delta f}{f_0} = \frac{\Delta f \lambda}{c} = \frac{8\times10^{6} \times 500\times10^{-9}}{3\times10^{8}} = \underline{1.3\times10^{-8}}$$

This is called the natural line width of the spectral line

Exercise #9

Use nonrelativistic Doppler formula $\lambda = 500\ nm$ at $T = 1000\ K$

$$\Delta E = \frac{3}{2} k_B T = 2\times10^{-20}\ J = \underline{0.129\ eV} = \frac{1}{2} m v^2 =$$

$\underline{\underline{v \ll c}}$

$$\frac{v}{c} = \frac{\Delta f}{f} = \frac{f' - f}{f} = \frac{\frac{1}{\lambda'} - \frac{1}{\lambda}}{\frac{1}{\lambda}} = \frac{\Delta f}{f}$$

$$f = \frac{c}{\lambda} = \frac{3\times10^8}{500\times10^{-9}} = 6\times10^{14}\ Hz$$

problems #16 in chapter 1

$$\Delta f = \frac{v}{c} f \qquad \frac{1}{2} m v^2 = \frac{3}{2} k_B T \Rightarrow v = \sqrt{\frac{3kT}{m}}$$

mass of the proton!!!

$$v = 4981\ m/s$$

$$\Delta f = \frac{4981}{3\times10^8} \times 6\times10^{14} = 9.962\times10^{9}\ Hz$$

$$f = \frac{c}{\lambda} \qquad df = -\frac{c}{\lambda^2} d\lambda$$

$$\Delta f = \frac{c}{\lambda^2} \Delta\lambda \Rightarrow \underline{\underline{\Delta\lambda = \Delta f \frac{\lambda^2}{c}}}$$

$$\Delta\lambda = 8.3\times10^{-12} = \underline{\underline{0.083\ \mathring{A}}}$$

Δp_x is less than $-20\ \frac{MeV}{c}$

and more than $20\ \frac{MeV}{c}$

$$-20\,MeV \le \Delta p_x \le 20\,MeV$$

nucleus

this is large momentum

It is relativistic $\Rightarrow E^2 = p^2c^2 + m_0^2c^4$

$\Delta p_x \geq 20\ \frac{MeV}{c}$

at least $p_x = 20\ \frac{MeV}{c}$

$$= \left(20\ \frac{MeV}{c}\right)^2 c^2 + (0.511\ MeV)^2$$

$$E^2 = 400 + 0.511\ (MeV)^2 = 400\ (MeV)^2$$

$$E \geq 20\ MeV$$

$$K = E - m_0c^2 \geq 19.5\,MeV$$

β decay

$${}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^-$$

A (mass #) = # of neutrons + protons

Z (charge #) = # of protons

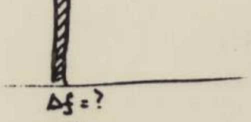
Electrons emitted in radioactive beta decays have energies much less than 19.5 MeV

Electrons observed in β decay do not come from within the nucleus but are actually created at the instant of decay.

$\boxed{\text{Ex 4.12 The width of spectral lines}}$

τ: average time after excitation at which a group of atoms radiates (lifetime) $\tau = 10^{-8}$ sec.

find $\Delta f = ?$ (the line width)



$\Delta f = ?$ or $\Delta\lambda$

use $\Delta E\, \Delta t \geq \frac{\hbar}{2}$

$\Delta E = h\,\Delta f \qquad \Delta t = 10^{-8}$ sec

$$h\,\Delta f\,\Delta t \geq \frac{h}{4\pi}$$

ΔE

$\Delta E = h\Delta f$

$$\Delta f = \frac{1}{4\pi\,\Delta t} = \frac{10^8}{4\pi} = 8 \times 10^6\ Hz$$

Ex 4.10

a) Consider a racketball ~~confined~~ of mass $m = 100\,g$ confined to a room 15m on a side. Assume the ball is moving at 2.0 m/s along the x-axis.

find $\frac{\Delta V_x}{V_x} = ?$

$$\Delta p_x \geq \frac{\hbar}{2\Delta x} = \frac{1.05\times 10^{-34}}{2\times 15} = 3.5\times 10^{-36}\ Kg\,\frac{m}{s}$$

The minimum spread in velocity is

$$\Delta V_x = \frac{\Delta p_x}{m} = 3.5\times 10^{-35}\ m/s$$

relative uncertainty

$$\frac{\Delta V_x}{V_x} = 1.8\times 10^{-35}$$ which is unmeasurable quantity.

b) If the ball were to suddenly move along the y-axis ...

$\Delta y = ?$ $t = 1\ sec$

also $\Delta V_y = 3.5\times 10^{-35}\ m/s$

$$\Delta y = \Delta V_y \cdot t = 3.5\times 10^{-35}\ m$$ in the y-direct.

This is also unmeasurable it is $\frac{1}{10^{20}}$ the size of the nucleus.

Ex 4.11 Do electrons exist within the nucleus?

Find the Kinetic energy of an electron confined within a nucleus?

$1.\times 10^{-14}\,m$ (size of the nucleus)

$$\Delta p_x \geq \frac{\hbar}{2\underbrace{\Delta x}_{\text{half the confinement length.}}} = \frac{1.05\times 10^{-34}}{1\times 10^{-14}} = 1.05\times 10^{-20}\ Kg\,\frac{m}{s}$$

$$\Delta p_x \geq 2.0\times 10^{7}\ \frac{eV}{c} = 20\ \frac{MeV}{c}$$

4.5 The Heisenberg Uncertainty Principle

The uncertainty relationships discussed before apply to all waves, and we should apply them to de Broglie waves.

$$p = \frac{h}{\lambda} \quad k = \frac{2\pi}{\lambda} \Rightarrow p = \frac{hk}{2\pi} = \hbar k \quad \left(\hbar = \frac{h}{2\pi}\right)$$

$$\hbar = 1.05 \times 10^{-34} \, J.s$$

So $\Delta k = \dfrac{\Delta p}{\hbar} \Rightarrow \quad \Delta x \, \Delta p_x \sim \hbar$

along the x-direction

$\Delta y \, \Delta p_y \sim \hbar$ and $\Delta z \, \Delta p_z \sim \hbar$

In your textbook you have $\Delta p_x \Delta x \geq \frac{\hbar}{2}$

"If a measurement of position is made with precision Δx and a simultaneous measurement of momentum in the x-dir. is made with a precision Δp_x, then the product of the two uncertainties can never be smaller than $\hbar/2$."

We know that $E = hf = \hbar \omega$

$\Rightarrow \Delta E = \hbar \Delta \omega \quad \Rightarrow \Delta \omega = \dfrac{\Delta E}{\hbar}$

$\Rightarrow \quad \Delta E * \Delta t \sim \hbar \geq \hbar/2$

ΔE is the uncertainty in the energy of the wave packet and Δt is time taken to measure that

Werner Heisenberg invented a complete theory of quantum mechanics called matrix mechanics (with the help of Born and Jordan). We discuss the uncertainty principle of Heisenberg (1927)

This is called "uncertainty principle" for classical waves.

One can find a similar uncertainty relationship betwen frequency and time so that $\Delta\omega * \Delta t \simeq 1$.

Example.

Water wave; 10 wave crests are counted in a distance of 200 cm. Estimate the minimum uncertainty in λ. That is $\Delta\lambda = ?$

$$k = \frac{2\pi}{\lambda} \qquad dk = -2\pi \frac{d\lambda}{\lambda^2}$$

$$\Rightarrow |\Delta k| = 2\pi \frac{\Delta\lambda}{\lambda^2}$$

use $\Delta k \, \Delta x \simeq 1$ $\qquad \Delta x \left(\frac{2\pi}{\lambda^2} \Delta\lambda \right) \sim 1$

$$\Delta x \, \Delta\lambda \sim \frac{\lambda^2}{2\pi}$$

$$\Delta\lambda \sim \frac{1}{\Delta x} \frac{\lambda^2}{2\pi} = \frac{1}{2} \frac{(0.2)^2}{2\pi} \sim \underline{\underline{0.3 \text{ cm}}}$$

$$\lambda = \frac{200}{10} = 20 \text{ cm} = 0.2 \text{ m}$$

$\Delta x = 200 \text{ cm} = \underline{\underline{2 \text{ m}}}$ — very large

$\Delta\lambda \approx \underline{\underline{0.3 \text{ cm}}}$ — small

Let us now look at an important difference between classical particles and waves.

Consider a wave $y = A\cos k_1 x$ (pure cosine wave)

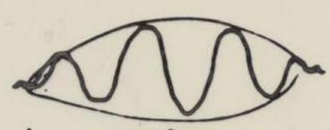
λ_1

$\Delta K = 0$
$\Delta x = \infty$

it expands from $-\infty$ to ∞. where is the wave located? It is everywhere. On the other hand, its wavelength is precisely determined. $\lambda = 2\pi/k_1$.

If we need to use a wave to represent a particle, it must be localized, or confined to a relatively small region of space

If we add many cosine waves with diff λ or k we could reach this situation



this is called a wave packet

$\longleftarrow \Delta x \longrightarrow$ extension of the wave

The range of wavelengths (wavenumbers) is denoted ΔK is large in this case while the wave can be located.

When ~~the~~ we had a single wave ΔK was zero and Δx was infinite. As we increased ΔK (by adding more waves) we decreased Δx (the wave became more confined).

We seem to have an inverse relationship between Δx and Δk; as one decreases, the other increases. An approximate mathematical relationship is $\Delta x \cdot \Delta k \approx 1$

Individual de Broglie waves representing a particle of mass m show dispersion. $v_p \geq c$!!! But:
What we are interested in is the wave group velocity

$$v_g = v_p + k \left.\frac{dv_p}{dk}\right|_{k_0}$$

$$E = hf = \hbar\omega$$

$$p = \frac{h}{\lambda} = \hbar k$$

$$= c\sqrt{1+\left(\frac{mc}{\hbar k_0}\right)^2} + \cdots$$

$$v_g = \frac{d\omega}{dk} = \frac{dE}{dp}$$

$$v_p = c\left[1+\left(\frac{mc}{\hbar}\right)^2\frac{1}{k^2}\right]^{1/2}$$

$$E = \sqrt{p^2c^2+m_0^2c^4}$$

$$\frac{dv_p}{dk} = c\left\{\frac{1}{2}\,(-)\,\frac{(mc)^2}{\hbar^2} \cdots\right.$$

$$\frac{dE}{dp} = \frac{2pc^2}{2\sqrt{p^2c^2+m_0^2c^4}} \quad (=E)$$

$$= \frac{\gamma m v c^2}{\gamma m c^2} = v$$

That corresponds to the motion of the body and $v_g < c$ as it should be.

$$v_g = \frac{c}{\left[1+\left(\frac{m_0 c}{\hbar k_0}\right)^2\right]^{1/2}} = \frac{c^2}{\left.v_p\right|_{k_0}}$$

but $v_p = \lambda f = \frac{E}{p} = \frac{\gamma m c^2}{\gamma m v} = \frac{c^2}{v}$ (v: particle velocity)

[v_p has no physical significance because its is the motion of the wave group, not the motion of the individual waves that make up the group,

$\Rightarrow v_g = v \Rightarrow$ the group velocity of the matter wave = particle speed!

The matter wave envelope should move at the speed of the particle!

An e^- has a de Broglie $\lambda = 2\times10^{-12}$ m, find K.E, v_p and v_g

a) $pc = \frac{hc}{\lambda} = 6.20\times10^5$ eV $= \frac{12400 \text{ eV Å}}{0.02 \text{ Å}} = 6.2\times10^5$ eV

$E_0 = 511$ keV $\qquad E = K + E_0 \Rightarrow K = E - E_0 = \sqrt{p^2c^2+E_0^2} - E_0$

$K = 292$ keV

b) $v_p = \frac{c^2}{v} = 1.30c$ (no physical meaning)

$v_g = v = c\sqrt{1-\frac{E_0^2}{E^2}} = 0.771c \qquad \left[\; E = \gamma m_0 c^2 = \gamma E_0 = \frac{E_0}{\sqrt{1-v^2/c^2}}\right.$

Ex 4.5 Group velocity in Deep water waves

Newton's showed that the phase velocity ~~of dey~~ deep water waves having wavelength λ is given by

$$v_p = \sqrt{\frac{g\lambda}{2\pi}}$$

find v_g? $\quad v_g = v_p\big|_{k_0} + k_0 \frac{dv_p}{dk}\Big|_{k_0}$

$$v_p = \sqrt{\frac{g}{k}} \qquad \frac{dv_p}{dk} = -\frac{1}{2}\left(\frac{g}{k}\right)^{-1/2}\left(+\frac{g}{k^2}\right)^{+1} = -\frac{1}{2k}\sqrt{\frac{g}{k}}$$

$$v_g = \sqrt{\frac{g}{k_0}} - \cancel{k_0}\,\frac{1}{2\cancel{k_0}}\sqrt{\frac{g}{k_0}} = \frac{1}{2}\sqrt{\frac{g}{k_0}} = \frac{v_p}{2}\Big|_{k_0}$$

$$v_p = \left(\frac{g}{k}\right)^{1/2} \quad \frac{dv_p}{dk} = \frac{1}{2}\left(-\frac{g}{k^2}\right)\left(\frac{g}{k}\right)^{-1/2} = -\frac{1}{2k}\sqrt{\frac{g}{k}}$$

* Matter Wave Packets

let us apply our general theory to electrons. let us show that . . .

According to de Broglie, individual matter waves have f and λ given by

$$f = \frac{E}{h}\ \text{(energy of the electron)} \quad \text{and} \quad \lambda = \frac{h}{p}\ \text{(momentum of the electron)}$$

the phase speed of these matter waves is $v_p = \lambda f = \dfrac{E}{p}$

but $E = \sqrt{p^2c^2 + m_0^2c^4} = pc\sqrt{1 + \dfrac{m_0^2c^2}{p^2}}$

$$v_p = c\sqrt{1 + \left(\frac{m_0 c}{\hbar k}\right)^2} \qquad \left(\text{because } p = \frac{h}{\lambda} = \frac{hk}{2\pi} = \hbar k\right)$$

In the general case, many waves having continuous distribution of wavelengths must be added to form a packet that is finite over a limited range and zero everywhere else. In this case

$$\boxed{v_g = \frac{d\omega}{dk}\bigg|_{k_0}}$$

where k_0 is the central wavenumber of the many waves present

$k = 7, 8, 9, 10, 11, 12, 13$ $\qquad$ ΔK is called the spread of wavenumber $= 13-7=6$

$\uparrow$ $k_0 = 10$

Remember $\omega = k v_p \Rightarrow \boxed{v_g = v_p\big|_{k_0} + k \frac{dv_p}{dk}\bigg|_{k_0}}$

$v_p(k, \lambda)$

Materials in which the phase velocity varies with wavelength are said to exhibit "dispersion". An example is glass in which $n(\lambda)$ and different colors of light travel at diff. speeds.

If $v_p \neq (k, \lambda)$ ~~are~~ these materials are called "nondispersive"

In a nondispersive medium, where all waves have the same velocity, $v_g = v_p$. In a dispersive medium $v_g > v_p$ or $v_g < v_p$ depending on the sign of $\frac{dv_p}{dk}\bigg|_{k_0}$.

Ex 4.4. Group velocity in a dispersive medium

$v_p = \frac{A'}{\lambda} = Ak$ $\quad$ (A & A' are const.) $\qquad \lambda = \frac{2\pi}{k}$

$= \left(\frac{A'}{2\pi}\right)k = Ak$)

$$v_g = v_p\big|_{k_0} + k\frac{dv_p}{dk}\bigg|_{k_0} = Ak_0 + k_0 A = 2Ak_0 = 2v_p\big|_{k_0}$$

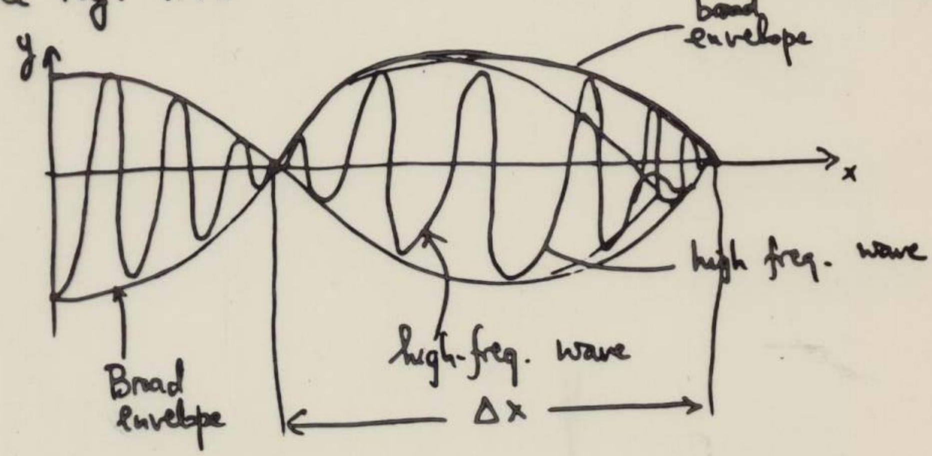
$$\text{So } y = \underbrace{2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)}_{\text{broad sinusoidal envelope}} \cdot \underbrace{\cos\left(k_1 x - \omega_1 t\right)}_{}$$

$$\frac{k_1+k_2}{2} \simeq k_1 \simeq k_2$$

$$\frac{\omega_1+\omega_2}{2} \simeq \omega_1 \simeq \omega_2$$

broad sinusoidal envelope modulating a high freq. wave within the envelope

See Fig. 4.12



For the wave within the envelope

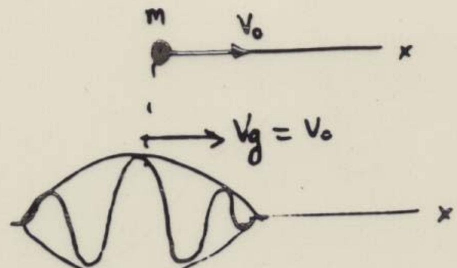
$$v_p = \frac{(\omega_1+\omega_2)/2}{(k_1+k_2)/2} \simeq \frac{\omega_1}{k_1} \equiv v_1 \simeq v_2$$

The wave moves at the phase velocity of one of the two waves.

For The envelope

$$v_g = \frac{(\omega_1-\omega_2)/2}{(k_1-k_2)/2} = \frac{\Delta\omega}{\Delta k}$$

It moves with a different velocity.



A wave group consists of a superposition of waves with "different wavelengths" as shown in Fig. 4.11. This is called called physically "beats" in case of sound waves ~~in the case~~ when two waves of ~~dif~~ slightly diff λ are combined.

Let us examine the same situation mathematically

Consider a one dim- sinusoidal wave

$$y = A \cos\left(\frac{2\pi}{\lambda}x - 2\pi f t\right) = A \cos(kx - \omega t)$$

$k = \frac{2\pi}{\lambda}$, $\omega = 2\pi f$ and $v_p = \frac{\omega}{k} = \lambda f$ is the "phase velocity". It is the velocity of the wave. This wave extends from $-\infty$ to $+\infty$ This wave, of course, cannot represent a localized p

Suppose now we have two waves of slightly diff wavelengths

The resultant wave is

$$y = y_1 + y_2 = A\cos(k_1 x - \omega_1 t) + A\cos(k_2 x - \omega_2 t)$$

$$y = 2A \cos\left[\frac{1}{2}\left\{\underbrace{(k_2 - k_1)}_{\Delta k} x - \underbrace{(\omega_2 - \omega_1)}_{\Delta\omega} t\right\}\right] \cdot \cos\left[\frac{1}{2}\left\{(k_1 + k_2)x - (\omega_1 + \omega_2)t\right]\right.$$

If the two waves have slightly different values of k and ω,

$\Delta k = k_2 - k_1$ is small ($\Delta\lambda$ is large)

and $\Delta\omega = \omega_2 - \omega_1$ is small (Δf is small)

Exercise 2 Monochromatic Neutrons

See fig. 4.9

a) $\lambda = 1.0$ Å $\quad p = mV = \frac{h}{\lambda}$

$$\Rightarrow V = \frac{h}{m_n \lambda} = \frac{6.63\times10^{-34}}{1\times10^{-10}\times 1.66\times10^{-27}}$$

$$= 3.99\times10^{3}\ \text{m/s}$$

b)

$$\theta = \omega t = 10^\circ = 0.175\ \text{rad}$$

$$t = \frac{x}{v} = \frac{0.5}{3.99\times10^{3}} = 1.25\times10^{-4}\ \text{sec}$$

$$\omega = \frac{\theta}{t} = \frac{0.175}{1.25\times10^{-4}} = 13965\ \text{rad/sec}$$

$$= 13963\times\frac{60}{2\pi} = 13336\ \text{rpm}.$$

4.3 Wave Groups and Dispersion (Wave velocity and group velocity of waves)

The matter wave representing a moving particle should be a pulse or "wave group" of limited spacial extent.

If we add a large # of sinusoidal with diff. wavelengths we get a pulse.

The wave group can be shown to move with a speed v_g (group speed) identical to the classical particle speed.

(See Fig 4.10 transparency)