Pb # 4.

$$\psi(x,y) = \psi_1(x)\,\psi_2(y)$$

$$= A \sin(k_1 x)\sin(k_2 y) \qquad k_1 = \frac{n_1\pi}{L} \quad k_2 = \frac{n_2\pi}{L}$$

$$E = \frac{\hbar^2\pi^2}{2mL^2}(n_1^2 + n_2^2)$$

$$E_{11} = \frac{\pi^2\hbar^2}{mL^2} \qquad \psi_{11} = A \sin\left(\frac{\pi}{L}x\right)\sin\left(\frac{\pi}{L}y\right)$$

$$E_{12} = \frac{5\pi^2\hbar^2}{2mL^2} = E_{21} \qquad \psi_{12} = A\sin\left(\frac{\pi}{L}x\right)\sin\left(\frac{2\pi}{L}y\right)$$

$$\psi_{21} = A\sin\left(\frac{2\pi}{L}x\right)\sin\left(\frac{\pi}{L}y\right)$$

$$A = \frac{2}{L}$$

Pb # 7.

$$\psi(x,y,z) = A\,\psi(x)\,\psi(y)\,\psi(z)$$

$$= A \sin(k_1 x)\sin(k_2 y)\sin(k_3 z)$$

$$1 = \int_0^L \psi^2\, dV \Rightarrow$$

$$\underbrace{A^2\int_0^L \sin^2(k_1 x)\,dx}_{A^2\left(\frac{L}{2}\right)}\ \underbrace{\int_0^L \sin^2(k_2 y)\,dy}_{\left(\frac{L}{2}\right)}\ \underbrace{\int_0^L \sin^2(k_3 z)\,dz}_{\left(\frac{L}{2}\right)} = 1$$

$$\boxed{A = \left(\frac{2}{L}\right)^{3/2}}$$

$$\underbrace{A^2\int_0^{L_1} \sin^2(k_1 x)\,dx}_{A^2\left(\frac{L_1}{2}\right)}\ \underbrace{\int_0^{L_2} \sin^2(k_2 y)\,dy}_{\left(\frac{L_2}{2}\right)}\ \underbrace{\int_0^{L_3} \sin^2(k_3 z)\,dz}_{\left(\frac{L_3}{2}\right)} = 1$$

$$A = \sqrt{\frac{8}{L_1L_2L_3}} = \sqrt{\frac{8}{V}} \qquad V = \text{volume} = L_1L_2L_3.$$

Pb # 9.

$L = 4.714 \times 10^{-34}$ J.s $= \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{\ell(\ell+1)}\,1.05 \times 10^{-34}$

$\Rightarrow$

$\ell(\ell+1) \simeq 20$

$\ell^2 + \ell - 20 = 0 \quad \ell = \dfrac{-1 \pm \sqrt{1+80}}{2} = \boxed{4}$

Pb # 13 $\quad He^+ \quad n = 3 \Rightarrow$ a) $\ell = 0 \quad m_\ell = 0$

$\ell = 1 \quad m_\ell = 1, 0, -1$

$\ell = 2 \quad m_\ell = 2, 1, 0, -1, -2$

b) $E = -\dfrac{13.6}{9}(2)^2 = \underline{-6.04\ eV}$

Pb # 19.

6g state

a) $n = 6$

b) $E = -\dfrac{13.6}{n^2}\ (eV) = -\dfrac{13.6}{64} = \boxed{-0.378\,eV}$

c) $\ell = 4 \Rightarrow L = \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{20}\,\hbar = \boxed{4.72 \times 10^{-34}\ J.s}$

d) $-\ell \le m_\ell \le \ell \Rightarrow m_\ell = 4, 3, 2, 1, 0, -1, -2, -3, -4.$

$L_z = m_\ell \hbar$

$m_\ell = 4 \quad L_z = 4\hbar = 4.22 \times 10^{-34} \quad \cos\theta = \dfrac{L_z}{|\vec{L}|} = \dfrac{m_\ell}{\sqrt{\ell(\ell+1)}}$

$\cos\theta = \dfrac{4}{\sqrt{20}} = 0.89 \Rightarrow \boxed{\theta = 26.6^\circ}$

$m_\ell = 3 \quad L_z = 3\hbar = 3.165 \times 10^{-34} \quad \cos\theta = \dfrac{3}{\sqrt{20}} = 0.671$

$\Rightarrow \boxed{\theta = 47.9^\circ}$

etc...

Pb# 23. $R_{2p}(r) = A r e^{-r/2a_0}$ $\quad P(r) = A^2 r^2 e^{-r/a_0}$

$$\langle r \rangle = \int_0^\infty r \; r^2 |R_{2p}(r)|^2 \, dr \qquad A = \left(\frac{1}{2a_0}\right)^{3/2} \frac{1}{\sqrt{3}\,a_0}$$

$$= A^2 \int_0^\infty r^5 e^{-r/a_0} \, dr$$

let $\frac{r}{a_0} = \alpha$ $\quad r^5 = \alpha^5 a_0^5$ $\quad dr = a_0 \, d\alpha$

$$\langle r \rangle = A^2 a_0^6 \int_0^\infty \alpha^5 e^{-\alpha} \, d\alpha = A^2 a_0^6 \; 5!$$

$$5! = 5 \times 4 \times 3 \times 2$$

$$A^2 = \frac{1}{(2a_0)^3} \frac{1}{3a_0^2} = \frac{1}{3 \times 2 \times a_0^5}$$

$2 \times 2 \times 2 = (2a_0)^3$

$$\langle r \rangle = 5 a_0 = 5 \times 0.53 \, \text{Å} = \boxed{2.65 \, \text{Å}}$$

Pb # 27.

$$P = \int_0^{4a_0} P_{2s}(r) \, dr = \int_0^{4a_0} r^2 |R_{2s}(r)|^2 \, dr$$

$$= \int_0^{4a_0} r^2 \left(\frac{1}{2a_0}\right)^3 \left(2 - \frac{r}{a_0}\right)^2 e^{-\frac{r}{a_0}} \, dr$$

$$= \frac{1}{8a_0} \int_0^{4a_0} \left(\frac{r}{a_0}\right)^2 \left(2 - \frac{r}{a_0}\right)^2 e^{-\frac{r}{a_0}} \, dr$$

let $z = \frac{r}{a_0} \Rightarrow r = z a_0 \quad dr = a_0 \, dz$

$$P = \frac{\cancel{a_0}}{8\cancel{a_0}} \int_0^4 z^2 (4 + z^2 - 4z) \, e^{-z} \, dz$$

$$= \frac{1}{8} \int_0^4 (4z^2 - 4z^3 + z^4) \, e^{-z} \, dz$$

$$= \frac{1}{8} \left[4 \int_0^4 z^2 e^{-z} \, dz - 4 \int_0^4 z^3 e^{-z} \, dz + \int_0^4 z^4 e^{-z} \, dz \right]$$

II