King Fahd University of Petroleum & Minerals

Physics Department

Phys212- Quiz#7

Name: Key ID#:

The electron in a hydrogen atom is in a state described by the wavefunction $\Psi_{322}(r)$.

(a) What is the magnitude of the orbital angular momentum of the electron in this state?

$$|\vec{L}| = \hbar\sqrt{\ell(\ell+1)}$$

$$\ell = 2 \Rightarrow \boxed{|\vec{L}| = \hbar\sqrt{6}}$$

(b) What is the angle between the angular momentum vector and the z-axis?

$$m_\ell = 2$$

$$\cos\theta = \frac{L_z}{|\vec{L}|} = \frac{m_\ell \hbar}{\sqrt{\ell(\ell+1)}\,\hbar} = \frac{m_\ell}{\sqrt{\ell(\ell+1)}} = \frac{2}{\sqrt{6}}$$

$$\boxed{\theta = 35.3^\circ}$$

(c) Calculate the average value of r for the electron in this state.

Given $\int_0^\infty x^n e^{-x} dx = n!$

$$\langle r \rangle = \int_0^\infty r \, r^2 |R_{32}|^2 dr$$

$$R_{32} = \left(\frac{1}{3a_0}\right)^{3/2} \frac{2\sqrt{2}}{27\sqrt{5}} \left(\frac{r}{a_0}\right)^2 e^{-\frac{r}{3a_0}}$$

$$R_{32}^2 = \left(\frac{1}{3a_0}\right)^3 \frac{8}{3645} \left(\frac{r}{a_0}\right)^4 e^{-\frac{2r}{3a_0}}$$

$$\langle r \rangle = \left(\frac{1}{3a_0}\right)^3 \left(\frac{1}{a_0}\right)^4 \frac{8}{3645} \int_0^\infty r^7 e^{-\frac{2r}{3a_0}} dr$$

let $\frac{2r}{3a_0} = z \qquad r = \frac{3a_0}{2} z \quad dr = \frac{3a_0}{2} dz$

$$\langle r \rangle = \frac{8}{98415} \frac{1}{a_0^7} \left(\frac{3a_0}{2}\right)^8 \underbrace{\int_0^\infty z^7 e^{-z} dz}_{5040} = 10.5\, a_0 = \boxed{5.6\,\text{Å}}$$