

Name:

Key

ID#:

1. Write the normalized wavefunction for the second excited state of the electron in an infinite one dimensional well of length L.

In general for an infinite one dimensional well the normalized wavefunction are $\Psi_n(x,t) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) e^{-i\omega t}$

For the second excited state $n=3$

$$\Psi_3(x,t) = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi x}{L}\right) e^{-i\omega t}$$

2. What is the energy of the electron in eV if $L = 5 \text{ nm}$?

In general $E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$

$$n=3 \quad E_3 = \frac{9 \pi^2 \hbar^2}{2mL^2} = \frac{9 \times \pi^2 \times (1.05 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (5 \times 10^{-9})^2} = 2.15 \times 10^{-20} \text{ J} = \boxed{0.135 \text{ eV}}$$

3. What is the probability of finding this electron in the region between $L=0$ and $L=1 \text{ nm}$?

$$P = \int_0^{L/5} |\Psi_3|^2 dx = \frac{2}{L} \int_0^{L/5} \sin^2\left(\frac{3\pi x}{L}\right) dx$$

$$= \frac{2}{L} \int_0^{L/5} \frac{1}{2} (1 - \cos\left(\frac{6\pi x}{L}\right)) dx$$

$$= \frac{1}{L} \left\{ x \Big|_0^{L/5} - \frac{L}{6\pi} \sin\left(\frac{6\pi x}{L}\right) \Big|_0^{L/5} \right\}$$

$$= \frac{1}{L} \left\{ \frac{L}{5} - \frac{L}{6\pi} \left(\sin \frac{6\pi}{5} \right) - 0 \right\}$$

$$= 0.231 = \boxed{23.1\%}$$