Physics 212 – Quiz #5
Chapter 5

27 April 2004
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Name: Key Id#:

(a) Write the normalized wavefunction for the second excited state of an electron in an infinite on dimensional well of length L.

In general, for an infinite one dimensional well the normalized wavefunctions are $\Psi_n(x,t) = \sqrt{\frac{2}{L}} \sin\left(n\frac{\pi}{L}x\right) e^{-i\omega t}$

for the second excited state $n = 3$ $\boxed{\Psi_3(x,t) = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi}{L}x\right) e^{-i\omega t}}$

(b) What is the energy of this electron in (eV) if L = 5 nm?

In general $E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$

$n=3 \Rightarrow E_3 = \frac{9\pi^2\hbar^2}{2mL^2} = \frac{9 \times (\pi)^2 \times (1.05\times10^{-34})^2}{2 \times 9.1\times10^{-31} \times (5\times10^{-9})^2} = 2.15\times10^{-20}\ J$

$= \boxed{0.135\ eV}$

(c) What is the probability of finding this electron in the region between L = 0 and L = 1 nm?

$$P = \int_0^{\frac{L}{5}} |\Psi(x)|^2 dx = \frac{2}{L}\int_0^{\frac{L}{5}} \sin^2\left(\frac{3\pi}{L}x\right) dx$$

$$= \frac{2}{L}\int_0^{\frac{L}{5}} \frac{1}{2}\left[1 - \cos\left(\frac{6\pi}{L}x\right)\right] dx$$

$$= \frac{1}{L}\left[x\Big|_0^{\frac{L}{5}} - \frac{L}{6\pi}\sin\left(\frac{6\pi}{L}x\right)\Big|_0^{\frac{L}{5}} \right]$$

$$= \frac{1}{L}\left[\frac{L}{5} - \left(\frac{L}{6\pi} * \sin\left(\frac{6\pi}{5}\right) - 0\right)\right]$$

$$= \frac{1}{5} - \frac{1}{6\pi}\sin\left(\frac{6\pi}{5}\right) = 0.231 = 23.1\ \%$$