

Name:

Key

ID#:

1. (a) At what speed does the kinetic energy of an electron equal its rest energy?

$$K = (\gamma - 1) m_e c^2 = 1 m_e c^2$$

$$\Rightarrow \gamma = 2 = \frac{1}{\sqrt{1 - v^2/c^2}} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{4}$$

$$\frac{v^2}{c^2} = \frac{3}{4} \Rightarrow v = \frac{\sqrt{3}}{2} c = \boxed{0.866 c \text{ m/s}}$$

- (b) What is the total energy in MeV?

$$E = \gamma m_e c^2 = 2 \times 0.511 = \boxed{1.022 \text{ MeV}}$$

2. The free neutron is known to decay into a proton, an electron, and a neutrino $\bar{\nu}$ (of negligible rest mass) according to the equation $n \rightarrow p + e + \bar{\nu}$. This is called **beta decay**.

How much energy, in MeV, is released as the result of this decay?

The masses of the proton, neutron and electron are 1.007276u and 1.008665u and 0.0005485u, respectively. $1u = 1.66 \times 10^{-27} \text{ kg}$.

$$\begin{aligned} \Delta M &= m_n - (m_p + m_e + m_{\bar{\nu}}) = u(1.008665 - 1.007276 - 0.0005485) \\ &= 8.405 \times 10^{-4} u = 8.405 \times 10^{-4} \left(931.5 \frac{\text{MeV}}{c^2} \right) = 0.783 \frac{\text{MeV}}{c^2} \end{aligned}$$

$$E = \Delta M c^2 = 0.783 \frac{\text{MeV}}{c^2} \times c^2 = \boxed{0.783 \text{ MeV}}$$