mous quote of Heisenberg: "The invisible elementary particle of modern physics does not have the property of occupying space any more than it has properties like color or solidity. Fundamentally, it is not a material structure in space and time but only a symbol that allows the laws of nature to be expressed in especially simple form."

Are you satisfied with viewing science as a set of predictive rules or do you prefer to see science as a description of an objective world of things—in the case of particle physics, tiny, scaled-down things? What problems are associated with each point of view?

PROBLEMS

5.1 The Pilot Waves of de Broglie

1. Calculate the de Broglie wavelength for a proton moving with a speed of 10^6 m/s.
2. Calculate the de Broglie wavelength for an electron with kinetic energy (a) 50 eV and (b) 50 keV.
3. Calculate the de Broglie wavelength of a 74-kg person who is running at a speed of 5.0 m/s.
4. The "seeing" ability, or resolution, of radiation is determined by its wavelength. If the size of an atom is of the order of 0.1 nm, how fast must an electron travel to have a wavelength small enough to "see" an atom?
5. To "observe" small objects, one measures the diffraction of particles whose de Broglie wavelength is approximately equal to the object's size. Find the kinetic energy (in electron volts) required for electrons to resolve (a) a large organic molecule of size 10 nm, (b) atomic features of size 0.10 nm, and (c) a nucleus of size 10 fm. Repeat these calculations using alpha particles in place of electrons.
6. An electron and a photon each have kinetic energy equal to 50 keV. What are their de Broglie wavelengths?
7. Calculate the de Broglie wavelength of a proton that is accelerated through a potential difference of 10 MV.
8. Show that the de Broglie wavelength of an electron accelerated from rest through a small potential difference V is given by $\lambda = 1.226/\sqrt{V}$, where λ is in nanometers and V is in volts.
9. Find the de Broglie wavelength of a ball of mass 0.20 kg just before it strikes the Earth after being dropped from a building 50 m tall.
10. An electron has a de Broglie wavelength equal to the diameter of the hydrogen atom. What is the kinetic energy of the electron? How does this energy compare with the ground-state energy of the hydrogen atom?
11. For an electron to be confined to a nucleus, its de Broglie wavelength would have to be less than 10^{-14} m. (a) What would be the kinetic energy of an electron confined to this region? (b) On the basis of this result, would you expect to find an electron in a nucleus? Explain.
12. Through what potential difference would an electron have to be accelerated to give it a de Broglie wavelength of 1.00×10^{-10} m?

5.2 The Davisson–Germer Experiment

13. Figure P5.13 shows the top three planes of a crystal with planar spacing d. If $2d \sin \theta = 1.01\lambda$ for the two waves shown, and high-energy electrons of wavelength λ penetrate many planes deep into the crystal, which atomic plane produces a wave that cancels the surface reflection? This is an example of how extremely narrow maxima in high-energy electron diffraction are formed—that is, there are no diffracted beams unless $2d \sin \theta$ is equal to an integral number of wavelengths.

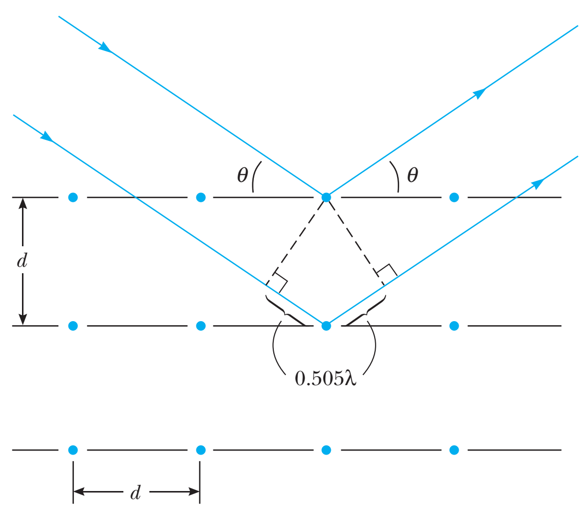

Figure P5.13

14. (a) Show that the formula for low-energy electron diffraction (LEED), when electrons are incident perpendicular to a crystal surface, may be written as

$$\sin \phi = \frac{nhc}{d(2m_e c^2 K)^{1/2}}$$

where n is the order of the maximum, d is the atomic spacing, m_e is the electron mass, K is the electron's kinetic energy, and ϕ is the angle between the incident and diffracted beams. (b) Calculate the atomic spacing in a crystal that has consecutive diffraction maxima at $\phi = 24.1°$ and $\phi = 54.9°$ for 100-eV electrons.

5.3 Wave Groups and Dispersion

15. Show that the group velocity for a nonrelativistic free electron is also given by $v_g = p/m_e = v_0$, where v_0 is the electron's velocity.
16. When a pebble is tossed into a pond, a circular wave pulse propagates outward from the disturbance. If you are alert (and it's not a sleepy afternoon in late August), you will see a fine structure in the pulse consisting of surface ripples moving inward through the circular disturbance. Explain this effect in terms of group and phase velocity if the phase velocity of ripples is given by $v_p = \sqrt{2\pi S/\lambda\rho}$, where S is the surface tension and ρ is the density of the liquid.
17. The dispersion relation for free relativistic electron waves is

$$\omega(k) = \sqrt{c^2k^2 + (m_e c^2/\hbar)^2}$$

Obtain expressions for the phase velocity v_p and group velocity v_g of these waves and show that their product is a constant, independent of k. From your result, what can you conclude about v_g if $v_p > c$?

5.5 The Heisenberg Uncertainty Principle

18. A ball of mass 50 g moves with a speed of 30 m/s. If its speed is measured to an accuracy of 0.1%, what is the minimum uncertainty in its position?
19. A proton has a kinetic energy of 1.0 MeV. If its momentum is measured with an uncertainty of 5.0%, what is the minimum uncertainty in its position?
20. We wish to measure simultaneously the wavelength and position of a photon. Assume that the wavelength measurement gives $\lambda = 6000$ Å with an accuracy of one part in a million, that is, $\Delta\lambda/\lambda = 10^{-6}$. What is the minimum uncertainty in the position of the photon?
21. A woman on a ladder drops small pellets toward a spot on the floor. (a) Show that, according to the uncertainty principle, the miss distance must be at least

$$\Delta x = \left(\frac{\hbar}{2m}\right)^{1/2}\left(\frac{H}{2g}\right)^{1/4}$$

where H is the initial height of each pellet above the floor and m is the mass of each pellet. (b) If $H = 2.0$ m and $m = 0.50$ g, what is Δx?
22. A beam of electrons is incident on a slit of variable width. If it is possible to resolve a 1% difference in momentum, what slit width would be necessary to resolve the interference pattern of the electrons if their kinetic energy is (a) 0.010 MeV, (b) 1.0 MeV, and (c) 100 MeV?
23. Suppose Fuzzy, a quantum-mechanical duck, lives in a world in which $h = 2\pi$ J·s. Fuzzy has a mass of 2.0 kg and is initially known to be within a region 1.0 m wide. (a) What is the minimum uncertainty in his speed? (b) Assuming this uncertainty in speed to prevail for 5.0 s, determine the uncertainty in position after this time.
24. An electron of momentum p is at a distance r from a stationary proton. The system has a kinetic energy $K = p^2/2m_e$ and potential energy $U = -ke^2/r$. Its total energy is $E = K + U$. If the electron is bound to the proton to form a hydrogen atom, its average position is at the proton but the uncertainty in its position is approximately equal to the radius, r, of its orbit. The electron's average momentum will be zero, but the uncertainty in its momentum will be given by the uncertainty principle. Treat the atom as a one-dimensional system in the following: (a) Estimate the uncertainty in the electron's momentum in terms of r. (b) Estimate the electron's kinetic, potential, and total energies in terms of r. (c) The actual value of r is the one that minimizes the total energy, resulting in a stable atom. Find that value of r and the resulting total energy. Compare your answer with the predictions of the Bohr theory.
25. An excited nucleus with a lifetime of 0.100 ns emits a γ ray of energy 2.00 MeV. Can the energy width (uncertainty in energy, ΔE) of this 2.00-MeV γ emission line be directly measured if the best gamma detectors can measure energies to ± 5 eV?
26. Typical measurements of the mass of a subatomic delta particle ($m \approx 1230$ MeV/c^2) are shown in Figure P5.26. Although the lifetime of the delta is much too short to measure directly, it can be calculated from the energy–time uncertainty principle. Estimate the lifetime from the full width at half-maximum of the mass measurement distribution shown.

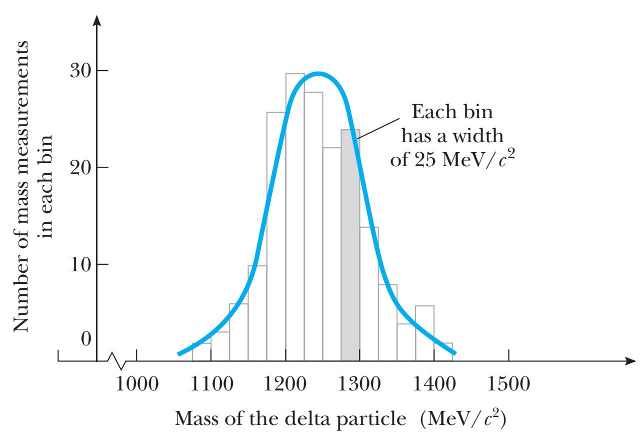

Figure P5.26 Histogram of mass measurements of the delta particle.

5.7 The Wave–Particle Duality

27. A monoenergetic beam of electrons is incident on a single slit of width 0.50 nm. A diffraction pattern is

formed on a screen 20 cm from the slit. If the distance between successive minima of the diffraction pattern is 2.1 cm, what is the energy of the incident electrons?

28. A neutron beam with a selected speed of 0.40 m/s is directed through a double slit with a 1.0-mm separation. An array of detectors is placed 10 m from the slit. (a) What is the de Broglie wavelength of the neutrons? (b) How far off axis is the first zero-intensity point on the detector array? (c) Can we say which slit any particular neutron passed through? Explain.
29. A two-slit electron diffraction experiment is done with slits of *unequal* widths. When only slit 1 is open, the number of electrons reaching the screen per second is 25 times the number of electrons reaching the screen per second when only slit 2 is open. When both slits are open, an interference pattern results in which the destructive interference is not complete. Find the ratio of the probability of an electron arriving at an interference maximum to the probability of an electron arriving at an adjacent interference minimum. (*Hint:* Use the superposition principle).

Additional Problems

30. Robert Hofstadter won the 1961 Nobel prize in physics for his pioneering work in scattering 20-GeV electrons from nuclei. (a) What is the γ factor for a 20-GeV electron, where $\gamma = (1 - v^2/c^2)^{-1/2}$? What is the momentum of the electron in kg·m/s? (b) What is the wavelength of a 20-GeV electron and how does it compare with the size of a nucleus?
31. An air rifle is used to shoot 1.0-g particles at 100 m/s through a hole of diameter 2.0 mm. How far from the rifle must an observer be to see the beam spread by 1.0 cm because of the uncertainty principle? Compare this answer with the diameter of the Universe (2×10^{26} m).
32. An atom in an excited state 1.8 eV above the ground state remains in that excited state 2.0 μs before moving to the ground state. Find (a) the frequency of the emitted photon, (b) its wavelength, and (c) its approximate uncertainty in energy.
33. A π^0 meson is an unstable particle produced in high-energy particle collisions. It has a mass–energy equivalent of about 135 MeV, and it exists for an average lifetime of only 8.7×10^{-17} s before decaying into two γ rays. Using the uncertainty principle, estimate the fractional uncertainty $\Delta m/m$ in its mass determination.
34. (a) Find and sketch the spectral content of the rectangular pulse of width 2τ shown in Figure P5.34. (b) Show that a reciprocity relation $\Delta\omega\, \Delta t \approx \pi$ holds in this case. Take $\Delta t = \tau$ and define $\Delta\omega$ similarly. (c) What range of frequencies is required to compose a pulse of width $2\tau = 1$ μs? A pulse of width $2\tau = 1$ ns?

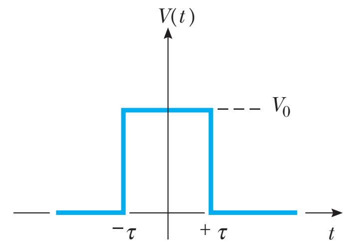

Figure P5.34

35. *A matter wave packet.* (a) Find and sketch the real part of the matter wave pulse shape $f(x)$ for a Gaussian amplitude distribution $a(k)$, where
$$a(k) = Ae^{-\alpha^2(k-k_0)^2}$$
Note that $a(k)$ is peaked at k_0 and has a width that decreases with increasing α. (*Hint:* In order to put $f(x) = (2\pi)^{-1/2}\int_{-\infty}^{+\infty} a(k)e^{ikx}\,dk$ into the standard form $\int_{-\infty}^{+\infty} e^{-az^2}\,dz$, complete the square in k.) (b) By comparing the result for the real part of $f(x)$ to the standard form of a Gaussian function with width Δx, $f(x) \propto Ae^{-(x/2\Delta x)^2}$, show that the width of the matter wave pulse is $\Delta x = \alpha$. (c) Find the width Δk of $a(k)$ by writing $a(k)$ in standard Gaussian form and show that $\Delta x\, \Delta k = \frac{1}{2}$, independent of α.
36. Consider a freely moving quantum particle with mass m and speed v. Its energy is $E = K + U = \frac{1}{2}mv^2 + 0$. Determine the phase speed of the quantum wave representing the particle and show that it is different from the speed at which the particle transports mass and energy.
37. In a vacuum tube, electrons are boiled out of a hot cathode at a slow, steady rate and accelerated from rest through a potential difference of 45.0 V. Then they travel altogether 28.0 cm as they go through an array of slits and fall on a screen to produce an interference pattern. Only one electron at a time will be in flight in the tube, provided the beam current is below what value? In this situation the interference pattern still appears, showing that each individual electron can interfere with itself.