PROBLEMS

4.2 The Composition of Atoms

1. Using the Faraday (96,500 C) and Avogadro's number, determine the electronic charge. Explain your reasoning.
2. *Weighing a copper atom in an electrolysis experiment.* A standard experiment involves passing a current of several amperes through a copper sulfate solution ($CuSO_4$) for a period of time and determining the mass of copper plated onto the cathode. If it is found that a current of 1.00 A flowing for 3600 s deposits 1.185 g of copper, find (a) the number of copper atoms deposited, (b) the weight of a copper atom, and (c) the molar mass of copper.
3. A mystery particle enters the region between the plates of a Thomson apparatus as shown in Figure 4.6. The deflection angle θ is measured to be 0.20 radians (downwards) for this particle when $V = 2000$ V, $\ell = 10.0$ cm, and $d = 2.00$ cm. If a perpendicular magnetic field of magnitude 4.57×10^{-2} T is applied simultaneously with the electric field, the particle passes through the plates without deflection. (a) Find q/m for this particle. (b) Identify the particle. (c) Find the horizontal speed with which the particle entered the plates. (d) Must we use relativistic mechanics for this particle?
4. Figure P4.4 shows a cathode ray tube for determining e/m_e without applying a magnetic field. In this case v_x may be found by measuring the rise in temperature when a known amount of charge is stopped in a target. If V, ℓ, d, D, and y are measured, e/m_e may be found. Show that

$$\frac{e}{m_e} = \frac{y v_x^2 d}{V\ell[(\ell/2) + D]}$$

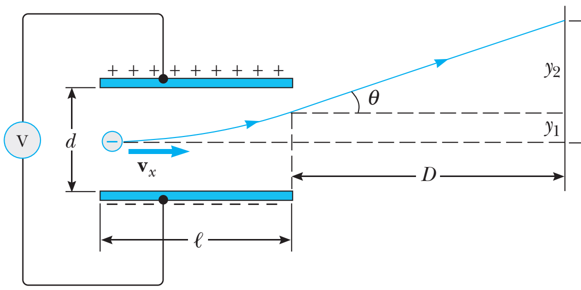

Figure P4.4 Deflection of a charged particle by an electric field.

5. *A Thomson-type experiment with relativistic electrons.* One of the earliest experiments to show that $p = \gamma mv$ (rather than $p = mv$) was that of Neumann. [G. Neumann, *Ann. Physik* 45:529 (1914)]. The apparatus shown in Figure P4.5 is identical to Thomson's except that the source of high-speed electrons is a radioactive radium source and the magnetic field $\mathbf{B}$ is arranged to act on the electron over its entire trajectory from source to detector. The combined electric and magnetic fields act as a *velocity selector*, only passing electrons with speed v, where $v = V/Bd$ (Equation 4.6), while in the region where there is only a magnetic field the electron moves in a circle of radius r, with r given by $p = Bre$. This latter region ($\mathbf{E} = 0$, $\mathbf{B} =$ constant) acts as a *momentum selector* because electrons with larger momenta have paths with larger radii. (a) Show that the radius of the circle described by the electron is given by $r = (l^2 + y^2)/2y$. (b) Typical values for the Neumann experiment were $d = 2.51 \times 10^{-4}$ m, $B = 0.0177$ T, and $l = 0.0247$ m. For $V = 1060$ V, y, the most critical value, was measured to be 0.0024 ± 0.0005 m. Show that these values disagree with the y value calculated from $p = mv$ but agree with the y value calculated from $p = \gamma mv$ within experimental error. (*Hint:* Find v from Equation 4.6, use $mv = Bre$ or $\gamma mv = Bre$ to find r, and use r to find y.)

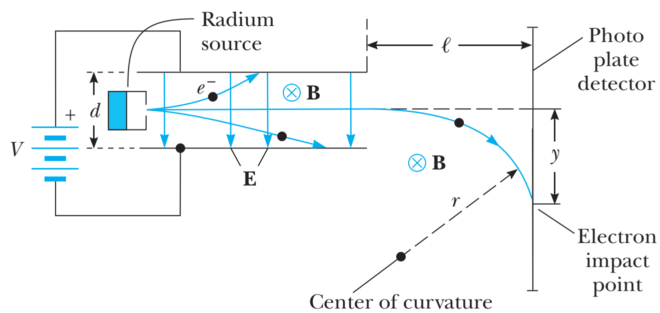

Figure P4.5 The Neumann apparatus.

6. In a Millikan oil-drop experiment, the condenser plates are spaced 2.00 cm apart, the potential across the plates is 4000 V, the rise or fall distance is 4.00 mm, the density of the oil droplets is 0.800 g/cm^3, and the viscosity of the air is 1.81×10^{-5} kg·m^{-1}s^{-1}. The average time of fall in the absence of an electric field is 15.9 s. The following different rise times in seconds are observed when the field is turned on: 36.0, 17.3, 24.0, 11.4, 7.54. (a) Find the radius and mass of the drop used in this experiment. (b) Calculate the charge on each drop, and show that charge is quantized by considering both the size of each charge and the amount of charge gained (lost) when the rise time changes. (c) Determine the electronic charge from these data. You may assume that e lies between 1.5 and 2.0×10^{-19} C.
7. Actual data from one of Millikan's early experiments are as follows:

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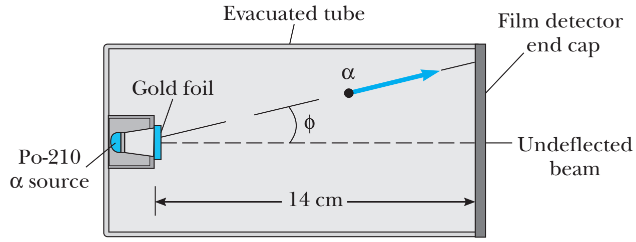
$a = 0.000276$ cm
$\rho = 0.9561$ g/cm^3
Average time of fall = 11.894 s
Rise or fall distance = 10.21 nm
Plate separation = 16.00 mm
Average potential difference between plates = 5085 V
Sequential rise times in seconds: 80.708, 22.336, 22.390, 22.368, 140.566, 79.600, 34.748, 34.762, 29.286, 29.236

Find the average value of e by requiring that the difference in charge for drops with different rise times be equal to an *integral* number of elementary charges.

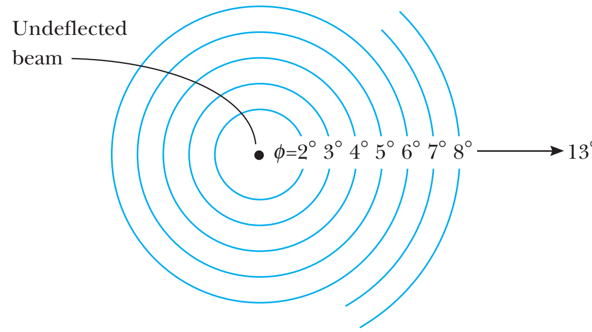
8. A parallel beam of α particles with fixed kinetic energy is normally incident on a piece of gold foil. (a) If 100 α particles per minute are detected at 20°, how many will be counted at 40°, 60°, 80°, and 100°? (b) If the kinetic energy of the incident α particles is doubled, how many scattered α particles will be observed at 20°? (c) If the original α particles were incident on a copper foil of the same thickness, how many scattered α particles would be detected at 20°? Note that $\rho_{Cu} = 8.9$ g/cm^3 and $\rho_{Au} = 19.3$ g/cm^3.
9. It is observed that α particles with kinetic energies of 13.9 MeV and higher, incident on Cu foils, do not obey Rutherford's $(\sin \phi/2)^{-4}$ law. Estimate the nuclear size of copper from this observation, assuming that the Cu nucleus remains fixed in a head-on collision with an α particle.
10. A typical Rutherford scattering apparatus consists of an evacuated tube containing a polonium-210 α source (5.2-MeV α's), collimators, a gold foil target, and a special alpha-detecting film. The detecting film simultaneously measures all the alphas scattered over a range from $\phi = 2.5°$ to $12.5°$. (See Fig. P4.10.) The total number of counts measured over a week's time falling in a specific ring (denoted by its average scattering angle) and the corresponding ring area are given in Table 4.2. (a) Find the counts per area at each angle and correct these values for the angle-independent background. The background correction may be found from a seven-day count taken with the beam blocked with a metal shutter in which 72 counts were measured evenly distributed over the total detector area of 8.50 cm^2. (b) Show that the corrected counts per unit area are proportional to $\sin^{-4}(\phi/2)$ or, in terms of the Rutherford formula, Equation 4.16,

$$\frac{\Delta n}{A} = \frac{C}{\sin^4(\phi/2)}$$

Notes: If a plot of $(\Delta n/A)$ versus ϕ will not fit on a single sheet of graph paper, try plotting $\log(\Delta n/A)$ versus $\log[1/(\sin \phi/2)^4]$. This plot should yield a straight line with a slope of 1 and an intercept that gives C. Explain why this technique works.



(a)



(b)

Figure P4.10 (a) Side view of Rutherford's scattering apparatus: ϕ is the scattering angle. (b) End view of the Rutherford apparatus showing the film detector end cap with grid marking the angle ϕ. The α particles damage the film emulsion and after development show up as dots within the rings.

Table 4.2 Data to Be Used in Problem 10

Angle (degrees)	Counts/Ring	Ring Area (cm^2)	Counts/Area
2.5	605	0.257	
3.5	631	0.360	
4.5	520	0.463	
5.5	405	0.566	
6.5	301	0.669	
7.5	201	0.772	
8.5	122	0.875	
9.5	78	0.987	
10.5	65	1.08	
11.5	66	1.18	
12.5	44	1.29	

4.3 The Bohr Atom

11. Calculate the wavelengths of the first three lines in the Balmer series for hydrogen.

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12. Calculate the wavelengths of the first three lines in the Lyman series for hydrogen.
13. (a) What value of n is associated with the Lyman series line in hydrogen whose wavelength is 102.6 nm? (b) Could this wavelength be associated with the Paschen or Brackett series?
14. (a) Use Equation 4.35 to calculate the radii of the first, second, and third Bohr orbits of hydrogen. (b) Find the electron's speed in the same three orbits. (c) Is a relativistic correction necessary? Explain.
15. (a) Construct an energy-level diagram for the He^+ ion, for which $Z = 2$. (b) What is the ionization energy for He^+?
16. Construct an energy level diagram for the Li^{2+} ion, for which $Z = 3$.
17. What is the radius of the first Bohr orbit in (a) He^+, (b) Li^{2+}, and (c) Be^{3+}?
18. A hydrogen atom initially in its ground state ($n = 1$) absorbs a photon and ends up in the state for which $n = 3$. (a) What is the energy of the absorbed photon? (b) If the atom returns to the ground state, what photon energies could the atom emit?
19. A photon is emitted from a hydrogen atom that undergoes an electronic transition from the state $n = 3$ to the state $n = 2$. Calculate (a) the energy, (b) the wavelength, and (c) the frequency of the emitted photon.
20. What is the energy of the photon that could cause (a) an electronic transition from the $n = 4$ state to the $n = 5$ state of hydrogen and (b) an electronic transition from the $n = 5$ state to the $n = 6$ state?
21. (a) Calculate the longest and shortest wavelengths for the Paschen series. (b) Determine the photon energies corresponding to these wavelengths.
22. Find the potential energy and kinetic energy of an electron in the ground state of the hydrogen atom.
23. A hydrogen atom is in its ground state ($n = 1$). Using the Bohr theory of the atom, calculate (a) the radius of the orbit, (b) the linear momentum of the electron, (c) the angular momentum of the electron, (d) the kinetic energy, (e) the potential energy, and (f) the total energy.
24. A hydrogen atom initially at rest in the $n = 3$ state decays to the ground state with the emission of a photon. (a) Calculate the wavelength of the emitted photon. (b) Estimate the recoil momentum of the atom and the kinetic energy of the recoiling atom. Where does this energy come from?
25. Calculate the frequency of the photon emitted by a hydrogen atom making a transition from the $n = 4$ to the $n = 3$ state. Compare your result with the frequency of revolution for the electron in these two Bohr orbits.
26. Calculate the longest and shortest wavelengths in the Lyman series for hydrogen, indicating the underlying electronic transition that gives rise to each. Are any of the Lyman spectral lines in the visible spectrum? Explain.
27. Show that Balmer's formula, $\lambda = C_2\left(\frac{n^2}{n^2 - 2^2}\right)$, reduces to the Rydberg formula, $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, provided that $(2^2/C_2) = R$. Check that $(2^2/C_2)$ has the same numerical value as R.
28. *The Auger process.* An electron in chromium makes a transition from the $n = 2$ state to the $n = 1$ state without emitting a photon. Instead, the excess energy is transferred to an outer electron (in the $n = 4$ state), which is ejected by the atom. (This is called an *Auger process*, and the ejected electron is referred to as an *Auger electron*.) Use the Bohr theory to find the kinetic energy of the Auger electron.
29. An electron initially in the $n = 3$ state of a one-electron atom of mass M at rest undergoes a transition to the $n = 1$ ground state. (a) Show that the recoil speed of the atom from emission of a photon is given approximately by

$$v = \frac{8hR}{9M}$$

(b) Calculate the percent of the $3 \rightarrow 1$ transition energy that is carried off by the recoiling atom if the atom is deuterium.
30. Apply classical mechanics to an electron in a stationary state of hydrogen to show that $L^2 = m_e k e^2 r$ and $L^3 = m_e k^2 e^4/\omega$. Here k is the Coulomb constant, L is the magnitude of the orbital angular momentum of the electron, and m_e, e, r, and ω are the mass, charge, orbit radius, and orbital angular frequency of the electron, respectively.
31. (a) Find the frequency of the electron's orbital motion, f_e, around a fixed nucleus of charge $+Ze$ by using Equation 4.24 and $f_e = (v/2\pi r)$ to obtain

$$f_e = \frac{m_e k^2 Z^2 e^4}{2\pi\hbar^3}\left(\frac{1}{n^3}\right)$$

(b) Show that the frequency of the photon emitted when an electron jumps from an outer to an inner orbit can be written

$$f_{\text{photon}} = \frac{kZ^2e^2}{2a_0 h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \frac{m_e k^2 e^4 Z^2}{2\pi\hbar^3}\left(\frac{n_i + n_f}{2n_i^2 n_f^2}\right)(n_i - n_f)$$

For an electronic transition between adjacent orbits, $n_i - n_f = 1$ and

$$f_{\text{photon}} = \frac{m_e k^2 Z^2 e^4}{2\pi\hbar^3}\left(\frac{n_i + n_f}{2n_i^2 n_f^2}\right)$$

Now examine the factor

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$$\left(\frac{n_i + n_f}{2n_i^2 n_f^2}\right)$$

and use $n_i > n_f$ to argue that

$$\frac{1}{n_i^3} < \frac{n_i + n_f}{2n_i^2 n_f^2} < \frac{1}{n_f^3}$$

(c) What do you conclude about the frequency of emitted radiation compared with the frequencies of orbital revolution in the initial and final states? What happens as $n_i \to \infty$?

32. Wavelengths of spectral lines depend to some extent on the nuclear mass. This occurs because the nucleus is not an infinitely heavy stationary mass and both the electron and nucleus actually revolve around their common center of mass. It can be shown that a system of this type is entirely equivalent to a single particle of reduced mass μ that revolves around the position of the heavier particle at a distance equal to the electron–nucleus separation. See Figure P4.32. Here, $\mu = m_e M/(m_e + M)$, where m_e is the electron mass and M is the nuclear mass. To take the moving nucleus into account in the Bohr theory we replace m_e with μ. Thus Equation 4.30 becomes

$$E_n = \frac{-\mu k e^2}{2m_e a_0}\left(\frac{1}{n^2}\right)$$

and Equation 4.33 becomes

$$\frac{1}{\lambda} = \frac{\mu k e^2}{2m_e a_0 hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \left(\frac{\mu}{m_e}\right) R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Determine the corrected values of wavelength for the first Balmer line ($n = 3$ to $n = 2$ transition) taking nuclear motion into account for (a) hydrogen, ^{1}H, (b) deuterium, ^{2}H, and (c) tritium, ^{3}H. (Deuterium, was actually discovered in 1932 by Harold Urey, who measured the small wavelength difference between ^{1}H and ^{2}H.)

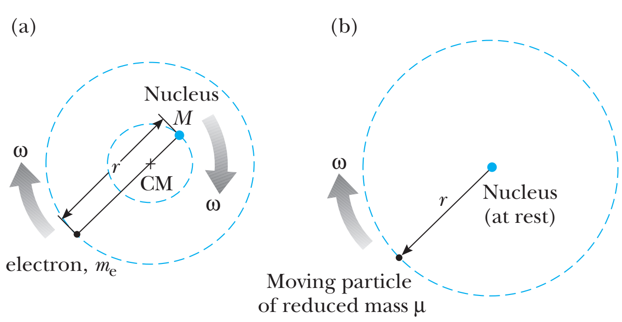

Figure P4.32 (a) Both the electron and the nucleus actually revolve around the center of mass. (b) To calculate the effect of nuclear motion, the nucleus can be considered to be at rest and m_e is replaced by the reduced mass μ.

33. A *muon* is a particle with a charge equal to that of an electron and a mass equal to 207 times the mass of an electron. Muonic lead is formed when ^{208}Pb captures a muon to replace an electron. Assume that the muon moves in such a small orbit that it "sees" a nuclear charge of $Z = 82$. According to the Bohr theory, what are the radius and energy of the ground state of muonic lead? Use the concept of reduced mass introduced in Problem 32.

34. A muon (Problem 33) is captured by a deuteron (an ^{2}H nucleus) to form a muonic atom. (a) Find the energy of the ground state and the first excited state. (b) What is the wavelength of the photon emitted when the atom makes a transition from the first excited state to the ground state? Use the concept of reduced mass introduced in Problem 32.

35. *Positronium* is a hydrogen-like atom consisting of a positron (a positively charged electron) and an electron revolving around each other. Using the Bohr model, find the allowed radii (relative to the center of mass of the two particles) and the allowed energies of the system. Use the concept of reduced mass introduced in Problem 32.

4.4 The Correspondence Principle

36. (a) Calculate the frequency of revolution and the orbit radius of the electron in the Bohr model of hydrogen for $n = 100$, 1000, and 10,000. (b) Calculate the photon frequency for transitions from the n to $n - 1$ states for the same values of n as in part (a) and compare with the revolution frequencies found in part (a). (c) Explain how your results verify the correspondence principle.

37. Use Bohr's model of the hydrogen atom to show that when the atom makes a transition from the state n to the state $n - 1$, the frequency of the emitted light is given by

$$f = \frac{2\pi^2 m_e k^2 e^4}{h^3}\left[\frac{2n - 1}{(n-1)^2 n^2}\right]$$

Show that as $n \to \infty$, the preceding expression varies as $1/n^3$ and reduces to the classical frequency one would expect the atom to emit. (*Hint:* To calculate the classical frequency, note that the frequency of revolution is $v/2\pi r$, where r is given by Equation 4.28.) This is an example of the correspondence principle, which requires that the classical and quantum models agree for large values of n.

4.5 The Franck–Hertz Experiment

38. An electron with kinetic energy less than 100 eV collides head-on in an elastic collision with a massive mercury atom at rest. (a) If the electron reverses direction in the collision (like a ball hitting a wall), show that the electron loses only a tiny fraction of its initial kinetic energy, given by

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$$\frac{\Delta K}{K} = \frac{4M}{m_e(1 + M/m_e)^2}$$

where m_e is the electron mass and M is the mercury atom mass. (b) Using the accepted values for m_e and M, show that

$$\frac{\Delta K}{K} \approx \frac{4m_e}{M}$$

and calculate the numerical value of $\Delta K/K$.

ADDITIONAL PROBLEMS

39. An electron collides inelastically and head-on with a mercury atom at rest. (a) If the separation of the first excited state and the ground state of the atom is exactly 4.9 eV, what is the minimum initial electron kinetic energy needed to raise the atom to its first excited state and also conserve momentum? Assume that the collision is completely inelastic. (b) What is the initial speed of the electron in this case? (c) What is the speed of the electron and atom after the collision? (d) What is the kinetic energy (in electron volts) of the electron after collision? Is the approximation that the electron loses all of its kinetic energy in an inelastic collision justified?
40. If the Franck–Hertz experiment could be performed with high-density monatomic hydrogen, at what voltage separations would the current dips appear? Take the separation between ground state and first excited state to be exactly 10.2 eV and be sure to consider momentum as well as energy in arriving at your answer.
41. Liquid oxygen has a bluish color, meaning that it preferentially absorbs light toward the red end of the visible spectrum. Although the oxygen molecule (O_2) does not strongly absorb visible radiation, it does absorb strongly at 1269 nm, which is in the infrared region of the spectrum. Research has shown that it is possible for two colliding O_2 molecules to absorb a single photon, sharing its energy equally. The transition that both molecules undergo is the same transition that results when they absorb 1269-nm radiation. What is the wavelength of the single photon that causes this double transition? What is the color of this radiation?
42. Two hydrogen atoms collide head-on and end up with zero kinetic energy. Each then emits a photon with a wavelength of 121.6 nm ($n = 2$ to $n = 1$ transition). At what speed were the atoms moving before the collision?
43 Steven Chu, Claude Cohen-Tannoudji, and William Phillips received the 1997 Nobel prize in physics for "the development of methods to cool and trap atoms with laser light." One part of their work was with a beam of atoms (mass $\sim 10^{-25}$ kg) that move at a speed on the order of 1 km/s, similar to the speed of molecules in air at room temperature. An intense laser light beam tuned to a visible atomic transition (assume 500 nm) is directed straight into the atomic beam. That is, the atomic beam and light beam are traveling in opposite directions. An atom in the ground state immediately absorbs a photon. Total system momentum is conserved in the absorption process. After a lifetime on the order of 10^{-8} s, the excited atom radiates by spontaneous emission. It has an equal probability of emitting a photon in any direction. Thus, the average "recoil" of the atom is zero over many absorption and emission cycles. (a) Estimate the average deceleration of the atomic beam. (b) What is the order of magnitude of the distance over which the atoms in the beam will be brought to a halt?
44. In a hot star, a multiply ionized atom with a single remaining electron produces a series of spectral lines as described by the Bohr model. The series corresponds to electronic transitions that terminate in the same final state. The longest and shortest wavelengths of the series are 63.3 nm and 22.8 nm, respectively. (a) What is the ion? (b) Find the wavelengths of the next three spectral lines nearest to the line of longest wavelength.

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