14. In the photoelectric effect, if the intensity of incident light is very low, then the number of photons per second striking the metal surface will be small and the probability per second of electron emission *per surface atom* will also be small. Account for the observed instantaneous emission of photoelectrons under these conditions.
15. *Blacker than black, brighter than white.* (a) Take a large, closed, empty cardboard box. Cut a slot a few millimeters wide in one side. Use black pens, markers, and soot to make some stripes next to the slot, as shown in Figure Q3.15a. Inspect them with care and choose which is blackest—the figure does not show enough contrast to reveal which it is. Explain why it is blackest. (b) Locate an intricately shaped compact fluorescent light fixture, as in Figure Q3.15b. Look at it through dark glasses and describe where it appears brightest. Explain why it is brightest there. *Suggestion:* Gustav Kirchhoff, professor at Heidelberg and master of the obvious, gave the same answer to part (a) as you likely will. His answer to part (b) would begin like this: When electromagnetic radiation falls on its surface, an object reflects some fraction r of the energy and absorbs the rest. Whether the fraction reflected is 0.8 or 0.001, the fraction absorbed is $a = 1 - r$. Suppose the object and its surroundings are at the same temperature. The energy the object absorbs joins its fund of internal energy, but the second law of thermodynamics implies that the absorbed energy cannot raise the object's temperature. It does not produce a temperature increase because the object's energy budget has one more term: energy radiated . . . . You still have to make the observations and answer questions (a) and (b), but you can incorporate some of Kirchhoff's ideas into your answer if you wish. *(Alexandra Héder)*

PROBLEMS

3.1 Light as an Electromagnetic Wave

1. *Classical Zeeman effect or the triumph of Maxwell's equations*! As pointed out in Section 3.1, Maxwell's equations may be used to predict the change in emission frequency when gas atoms are placed in a magnetic field. Consider the situation shown in Figure P3.1. Note that the application of a magnetic field perpendicular to the orbital plane of the electron induces an electric field, which changes the direction of the velocity vector. (a) Using

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

show that the magnitude of the electric field is given by

$$E = \frac{r}{2}\frac{dB}{dt}$$

(b) Using $F\,dt = m\,dv$, calculate the change in speed, Δv, of the electron. Show that if r remains constant,

$$\Delta v = \frac{erB}{2m_e}$$

(c) Find the change in angular frequency, $\Delta\omega$, of the electron and calculate the numerical value of $\Delta\omega$ for B equal to 1 T. Note that this is also the change in frequency of the light emitted according to Maxwell's equations. Find the fractional change in frequency, $\Delta\omega/\omega$, for an ordinary emission line of 500 nm. (d) Actually, the original emission line at ω_0 is split into three components at ω_0—$\Delta\omega$, ω_0, and $\omega_0 + \Delta\omega$. The line at $\omega_0 + \Delta\omega$ is produced by atoms with electrons rotating as shown in Figure P3.1, whereas the line at $\omega_0 - \Delta\omega$ is produced by atoms with electrons rotating in the opposite sense. The line at ω_0 is produced by atoms with electronic planes of rotation oriented parallel to $\mathbf{B}$. Explain.

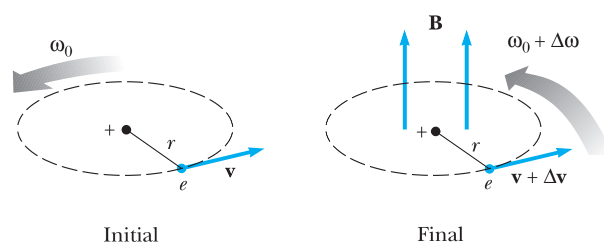

Figure P3.1

3.2 Blackbody Radiation

2. The temperature of your skin is approximately 35°C. What is the wavelength at which the peak occurs in the radiation emitted from your skin?
3. A 2.0-kg mass is attached to a massless spring of force constant $k = 25$ N/m. The spring is stretched 0.40 m from its equilibrium position and released. (a) Find the total energy and frequency of oscillation according to classical calculations. (b) Assume that the energy is quantized and find the quantum number, n, for the sys-

tem. (c) How much energy would be carried away in a 1-quantum change?

4. (a) Use Stefan's law to calculate the total power radiated per unit area by a tungsten filament at a temperature of 3000 K. (Assume that the filament is an ideal radiator.) (b) If the tungsten filament of a lightbulb is rated at 75 W, what is the surface area of the filament? (Assume that the main energy loss is due to radiation.)
5. Consider the problem of the distribution of blackbody radiation described in Figure 3.3. Note that as T *increases*, the wavelength $\lambda_{\max}$ at which $u(\lambda, T)$ reaches a maximum shifts toward shorter wavelengths. (a) Show that there is a general relationship between temperature and $\lambda_{\max}$ stating that $T\lambda_{\max}$ = constant (Wien's displacement law). (b) Obtain a numerical value for this constant. (*Hint:* Start with Planck's radiation law and note that the slope of $u(\lambda, T)$ is zero when $\lambda = \lambda_{\max}$.)
6. *Planck's fundamental constant, h.* Planck ultimately realized the great and fundamental importance of h, which, much more than a curve-fitting parameter, is actually the measure of all quantum phenomena. In fact, Planck suggested using the universal constants h, c (the velocity of light), and G (Newton's gravitational constant) to construct "natural" or universal units of length, time, and mass. He reasoned that the current units of length, time, and mass were based on the accidental size, motion, and mass of our particular planet, but that truly universal units should be based on the quantum theory, the speed of light in a vacuum, and the law of gravitation—all of which hold anywhere in the universe and at all times. Show that the expressions $\left(\frac{hG}{c^3}\right)^{1/2}$, $\left(\frac{hG}{c^5}\right)^{1/2}$, and $\left(\frac{hc}{G}\right)^{1/2}$ have dimensions of length, time, and mass and find their numerical values. These quantities are called, respectively, the Planck length, the Planck time, and the Planck mass. Would you care to speculate on the physical meaning of these quantities?

3.3 Derivation of the Rayleigh–Jeans Law and Planck's Law (Optional)

7. *Density of modes.* The essentials of calculating the number of modes of vibration of waves confined to a cavity may be understood by considering a one-dimensional example. (a) Calculate the number of modes (standing waves of different wavelength) with wavelengths between 2.0 cm and 2.1 cm that can exist on a string with fixed ends that is 2 m long. (*Hint:* use $n(\lambda/2) = L$, where $n = 1, 2, 3, 4, 5 \ldots$. Note that a specific value of n defines a specific mode or standing wave with different wavelength.) (b) Calculate, in analogy to our three-dimensional calculation, the number of modes per unit wavelength per unit length, $\frac{\Delta n}{L\,\Delta\lambda}$. (c) Show that in general the number of modes per unit wavelength per unit length for a string of length L is given by

$$\frac{1}{L}\left|\frac{dn}{d\lambda}\right| = \frac{2}{\lambda^2}$$

Does this expression yield the same numerical answer as found in (a)? (d) Under what conditions is it justified to replace $\left|\left(\frac{\Delta n}{L\,\Delta\lambda}\right)\right|$ with $\left|\left(\frac{dn}{L\,d\lambda}\right)\right|$? Is the expression $n = 2L/\lambda$ a continuous function?

3.4 Light Quantization and the Photoelectric Effect

8. Calculate the energy of a photon whose frequency is (a) 5×10^{14} Hz, (b) 10 GHz, (c) 30 MHz. Express your answers in electron volts.
9. Determine the corresponding wavelengths for the photons described in Problem 8.
10. An FM radio transmitter has a power output of 100 kW and operates at a frequency of 94 MHz. How many photons per second does the transmitter emit?
11. The average power generated by the Sun has the value 3.74×10^{26} W. Assuming the average wavelength of the Sun's radiation to be 500 nm, find the number of photons emitted by the Sun in 1 s.
12. A sodium-vapor lamp has a power output of 10 W. Using 589.3 nm as the average wavelength of the source, calculate the number of photons emitted per second.
13. The photocurrent of a photocell is cut off by a retarding potential of 2.92 V for radiation of wavelength 250 nm. Find the work function for the material.
14. The work function for potassium is 2.24 eV. If potassium metal is illuminated with light of wavelength 350 nm, find (a) the maximum kinetic energy of the photoelectrons and (b) the cutoff wavelength.
15. Molybdenum has a work function of 4.2 eV. (a) Find the cutoff wavelength and threshold frequency for the photoelectric effect. (b) Calculate the stopping potential if the incident light has a wavelength of 200 nm.
16. When cesium metal is illuminated with light of wavelength 300 nm, the photoelectrons emitted have a maximum kinetic energy of 2.23 eV. Find (a) the work function of cesium and (b) the stopping potential if the incident light has a wavelength of 400 nm.
17. Consider the metals lithium, beryllium, and mercury, which have work functions of 2.3 eV, 3.9 eV, and 4.5 eV, respectively. If light of wavelength 300 nm is incident on each of these metals, determine (a) which metals exhibit the photoelectric effect and (b) the

maximum kinetic energy for the photoelectron in each case.

18. Light of wavelength 500 nm is incident on a metallic surface. If the stopping potential for the photoelectric effect is 0.45 V, find (a) the maximum energy of the emitted electrons, (b) the work function, and (c) the cutoff wavelength.
19. The active material in a photocell has a work function of 2.00 eV. Under reverse-bias conditions (where the polarity of the battery in Figure 3.14 is reversed), the cutoff wavelength is found to be 350 nm. What is the value of the bias voltage?
20. A light source of wavelength λ illuminates a metal and ejects photoelectrons with a maximum kinetic energy of 1.00 eV. A second light source with half the wavelength of the first ejects photoelectrons with a maximum kinetic energy of 4.00 eV. What is the work function of the metal?
21. Figure P3.21 shows the stopping potential versus incident photon frequency for the photoelectric effect for sodium. Use these data points to find (a) the work function, (b) the ratio h/e, and (c) the cutoff wavelength. (d) Find the percent difference between your answer to (b) and the accepted value. (Data taken from R. A. Millikan, *Phys. Rev.*, 7:362, 1916.)

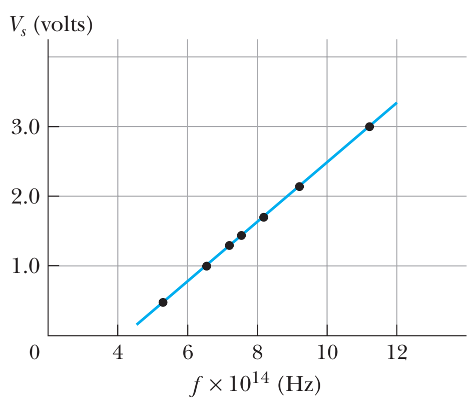

Figure P3.21 Some of Millikan's original data for sodium.

22. Photons of wavelength 450 nm are incident on a metal. The most energetic electrons ejected from the metal are bent into a circular arc of radius 20 cm by a magnetic field whose strength is equal to 2.0×10^{-5} T. What is the work function of the metal?

2.5 The Compton Effect and X-Rays

23. Calculate the energy and momentum of a photon of wavelength 500 nm.
24. X-rays of wavelength 0.200 nm are scattered from a block of carbon. If the scattered radiation is detected at 90° to the incident beam, find (a) the Compton shift, $\Delta\lambda$, and (b) the kinetic energy imparted to the recoiling electron.
25. X-rays with an energy of 300 keV undergo Compton scattering from a target. If the scattered rays are detected at 30° relative to the incident rays, find (a) the Compton shift at this angle, (b) the energy of the scattered x-ray, and (c) the energy of the recoiling electron.
26. X-rays with a wavelength of 0.040 nm undergo Compton scattering. (a) Find the wavelength of photons scattered at angles of 30°, 60°, 90°, 120°, 150°, 180°, and 210°. (b) Find the energy of the scattered electrons corresponding to these scattered x-rays. (c) Which one of the given scattering angles provides the electron with the greatest energy?
27. Show that a photon cannot transfer all of its energy to a free electron. (*Hint:* Note that energy and momentum must be conserved.)
28. In the Compton scattering event illustrated in Figure 3.24, the scattered photon has an energy of 120 keV and the recoiling electron has an energy of 40 keV. Find (a) the wavelength of the incident photon, (b) the angle θ at which the photon is scattered, and (c) the recoil angle ϕ of the electron.
29. Gamma rays (high-energy photons) of energy 1.02 MeV are scattered from electrons that are initially at rest. If the scattering is *symmetric*, that is, if $\theta = \phi$ in Figure 3.24, find (a) the scattering angle θ and (b) the energy of the scattered photons.
30. If the maximum energy given to an electron during Compton scattering is 30 keV, what is the wavelength of the incident photon? (*Hint:* What is the scattering angle for maximum energy transfer?)
31. A photon of initial energy 0.1 MeV undergoes Compton scattering at an angle of 60°. Find (a) the energy of the scattered photon, (b) the recoil kinetic energy of the electron, and (c) the recoil angle of the electron.
32. An excited iron (Fe) nucleus (mass 57 u) decays to its ground state with the emission of a photon. The energy available from this transition is 14.4 keV. (a) By how much is the photon energy reduced from the full 14.4 keV as a result of having to share energy with the recoiling atom? (b) What is the wavelength of the emitted photon?
33. Show that the Compton formula

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

results when expressions for the electron energy (Equation 3.33) and momentum (Equation 3.34) are substituted into the relativistic energy expression,

$$E_e^2 = p_e^2 c^2 + m_e^2 c^4$$

34. Find the energy of an x-ray photon that can impart a maximum energy of 50 keV to an electron by Compton collision.
35. Compton used photons of wavelength 0.0711 nm. (a) What is the energy of these photons? (b) What is the wavelength of the photons scattered at an angle of 180° (backscattering case)? (c) What is the energy of the backscattered photons? (d) What is the recoil energy of the electrons in this case?
36. A photon undergoing Compton scattering has an energy after scattering of 80 keV, and the electron recoils with an energy of 25 keV. (a) Find the wavelength of the incident photon. (b) Find the angle at which the photon is scattered. (c) Find the angle at which the electron recoils.
37. X-radiation from a molybdenum target (0.626 Å) is incident on a crystal with adjacent atomic planes spaced 4.00×10^{-10} m apart. Find the three smallest angles at which intensity maxima occur in the diffracted beam.
38. As a single crystal is rotated in an x-ray spectrometer (Fig. 3.22a), many parallel planes of atoms besides AA and BB produce strong diffracted beams. Two such planes are shown in Figure P3.38. (a) Determine geometrically the interplanar spacings d_1 and d_2 in terms of d_0. (b) Find the angles (with respect to the surface plane AA) of the $n = 1$, 2, and 3 intensity maxima from planes with spacing d_1. Let $\lambda = 0.626$ Å and $d_0 = 4.00$ Å. Note that a given crystal structure (for example, cubic) has interplanar spacings with characteristic ratios, which produce characteristic diffraction patterns. In this way, measurement of the angular position of diffracted x-rays may be used to infer the crystal structure.

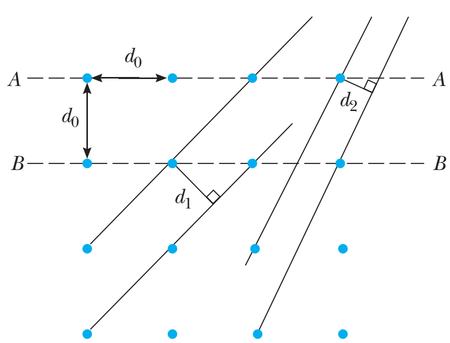

Figure P3.38 Atomic planes in a cubic lattice.

39. *The determination of Avogadro's number with x-rays.* X-rays from a molybdenum target (0.626 Å) are incident on an NaCl crystal, which has the atomic arrangement shown in Figure P3.39. If NaCl has a density of 2.17 g/cm^3 and the $n = 1$ diffraction maximum from planes separated by d is found at $\theta = 6.41°$, compute Avogadro's number. (*Hint:* First determine d. Using Figure P3.39, determine the number of NaCl molecules per primitive cell and set the mass per unit volume of the primitive cell equal to the density.)

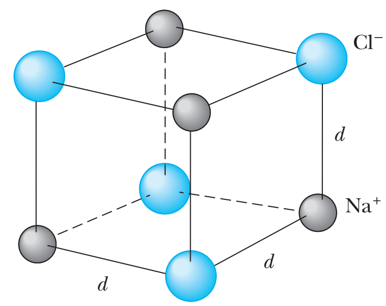

Figure P3.39 The primitive cell of NaCl.

3.7 Does Gravity Affect Light? (Optional)

40. In deriving expressions for the change in frequency of a photon falling or rising in a gravitational field, we have assumed a small change in frequency and a constant photon mass of hf/c^2. Suppose that a star is so dense that Δf is not small. (a) Show that f', the photon frequency at ∞, is related to f, the photon frequency at the star's surface, by

$$f' = fe^{-GM_s/R_s c^2}$$

(b) Show that this expression reduces to Equation 3.39 for small M_s/R_s. (*Hint:* The decrease in photon energy, $h\,df$, as the photon moves dr away from the star is equal to the work done against gravity, $F_G\,dr$.)
41. If the Sun were to contract and become a black hole, (a) what would its approximate radius be and (b) by what factor would its density increase?

Additional Problems

42. Two light sources are used in a photoelectric experiment to determine the work function for a particular metal surface. When green light from a mercury lamp ($\lambda = 546.1$ nm) is used, a retarding potential of 1.70 V reduces the photocurrent to zero. (a) Based on this measurement, what is the work function for this metal? (b) What stopping potential would be observed when using the yellow light from a helium discharge tube ($\lambda = 587.5$ nm)?
43. In a Compton collision with an electron, a photon of violet light ($\lambda = 4000$ Å) is backscattered through an angle of 180°. (a) How much energy (eV) is transferred to the electron in this collision? (b) Compare your result with the energy this electron would acquire in a photoelectric process with the same photon. (c) Could

violet light eject electrons from a metal by Compton collision? Explain.

44. Ultraviolet light is incident normally on the surface of a certain substance. The work function of the electrons in this substance is 3.44 eV. The incident light has an intensity of 0.055 W/m^2. The electrons are photoelectrically emitted with a maximum speed of 4.2×10^5 m/s. What is the maximum number of electrons emitted from a square centimeter of the surface per second? Pretend that none of the photons are reflected or heat the surface.

45. The following data are found for photoemission from calcium:

λ (*nm*)	253.6	313.2	365.0	404.7
V_s (*V*)	1.95	0.98	0.50	0.14

Plot V_s versus f, and from the graph obtain Planck's constant, the threshold frequency, and the work function for calcium.

46. A 0.500-nm x-ray photon is deflected through 134° in a Compton scattering event. At what angle (with respect to the incident beam) is the recoiling electron found?

47. An electron initially at rest recoils from a head-on collision with a photon. Show that the kinetic energy acquired by the electron is $2hf\alpha/(1 + 2\alpha)$, where α is the ratio of the photon's initial energy to the rest energy of the electron.

48. In a Compton scattering experiment, an x-ray photon scatters through an angle of 17.4° from a free electron that is initially at rest. The electron recoils with a speed of 2180 km/s. Calculate (a) the wavelength of the incident photon and (b) the angle through which the electron scatters.