SUGGESTIONS FOR FURTHER READING

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10. R. S. Shankland, "The Michelson-Morley Experiment," *Sci. Amer.*, November 1964, p. 107.
11. R. Skinner, *Relativity for Scientists and Engineers,* New York, Dover Publications, 1982.
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QUESTIONS

1. A particle is moving at a speed of less than $c/2$. If the speed of the particle is doubled, what happens to its momentum?
2. Give a physical argument showing that it is impossible to accelerate an object of mass m to the speed of light, even with a continuous force acting on it.
3. The upper limit of the speed of an electron is the speed of light, c. Does that mean that the momentum of the electron has an upper limit?
4. Because mass is a measure of energy, can we conclude that the mass of a compressed spring is greater than the mass of the same spring when it is not compressed?
5. Photons of light have zero mass. How is it possible that they have momentum?
6. "Newtonian mechanics correctly describes objects moving at ordinary speeds, and relativistic mechanics correctly describes objects moving very fast." "Relativistic mechanics must make a smooth transition as it reduces to Newtonian mechanics in a case where the speed of an object becomes small compared to the speed of light." Argue for or against each of these two statements.
7. Two objects are identical except that one is hotter than the other. Compare how they respond to identical forces.
8. With regard to reference frames, how does general relativity differ from special relativity?
9. Two identical clocks are in the same house, one upstairs in a bedroom, and the other downstairs in the kitchen. Which clock runs more slowly? Explain.
10. *A thought experiment.* Imagine ants living on a merry-go-round, which is their two-dimensional world. From measurements on small circles they are thoroughly familiar with the number π. When they measure the circumference of their world, and divide it by the diameter, they expect to calculate the number $\pi = 3.14159. . . .$ We see the merry-go-round turning at relativistic speed. From our point of view, the ants' measuring rods on the circumference are experiencing Lorentz contraction in the tangential direction; hence the ants will need some extra rods to fill that entire distance. The rods measuring the diameter, however, do not contract, because their motion is perpendicular to their lengths. As a result, the computed ratio does not agree with the number π. If you were an ant, you would say that the rest of the universe is spinning in circles, and your disk is stationary. What possible explanation can you then give for the discrepancy, in view of the general theory of relativity?

PROBLEMS

2.1 Relativistic Momentum and the Relativistic Form of Newton's Laws

1. Calculate the momentum of a proton moving with a speed of (a) $0.010c$, (b) $0.50c$, (c) $0.90c$. (d) Convert the answers of (a)–(c) to MeV/c.
2. An electron has a momentum that is 90% larger than its classical momentum. (a) Find the speed of the electron. (b) How would your result change if the particle were a proton?
3. Consider the relativistic form of Newton's second law. Show that when **F** is parallel to **v**,

$$F = m\left(1 - \frac{v^2}{c^2}\right)^{-3/2} \frac{dv}{dt}$$

where m is the mass of an object and v is its speed.

4. A charged particle moves along a straight line in a uniform electric field E with a speed v. If the motion and the electric field are both in the x direction, (a) show

that the magnitude of the acceleration of the charge q is given by

$$a = \frac{dv}{dt} = \frac{qE}{m}\left(1 - \frac{v^2}{c^2}\right)^{3/2}$$

(b) Discuss the significance of the dependence of the acceleration on the speed. (c) If the particle starts from rest at $x = 0$ at $t = 0$, find the speed of the particle and its position after a time t has elapsed. Comment on the limiting values of v and x as $t \to \infty$.

5. Recall that the magnetic force on a charge q moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ is equal to $q\mathbf{v} \times \mathbf{B}$. If a charged particle moves in a circular orbit with a fixed speed v in the presence of a constant magnetic field, use the relativistic form of Newton's second law to show that the frequency of its orbital motion is

$$f = \frac{qB}{2\pi m}\left(1 - \frac{v^2}{c^2}\right)^{1/2}$$

6. Show that the momentum of a particle having charge e moving in a circle of radius R in a magnetic field B is given by $p = 300BR$, where p is in MeV/c, B is in teslas, and R is in meters.

2.2 Relativistic Energy

7. Show that the energy–momentum relationship given by $E^2 = p^2c^2 + (mc^2)^2$ follows from the expressions $E = \gamma mc^2$ and $p = \gamma mu$.
8. A proton moves at a speed of $0.95c$. Calculate its (a) rest energy, (b) total energy, and (c) kinetic energy.
9. An electron has a kinetic energy 5 times greater than its rest energy. Find (a) its total energy and (b) its speed.
10. Find the speed of a particle whose total energy is 50% greater than its rest energy.
11. A proton in a high-energy accelerator is given a kinetic energy of 50 GeV. Determine the (a) momentum and (b) speed of the proton.
12. An electron has a speed of $0.75c$. Find the speed of a proton that has (a) the same kinetic energy as the electron and (b) the same momentum as the electron.
13. Protons in an accelerator at the Fermi National Laboratory near Chicago are accelerated to an energy of 400 times their rest energy. (a) What is the speed of these protons? (b) What is their kinetic energy in MeV?
14. *How long will the Sun shine, Nellie*? The Sun radiates about 4.0×10^{26} J of energy into space each second. (a) How much mass is released as radiation each second? (b) If the mass of the Sun is 2.0×10^{30} kg, how long can the Sun survive if the energy release continues at the present rate?
15. Electrons in projection television sets are accelerated through a total potential difference of 50,000 V. (a) Calculate the speed of the electrons using the relativistic form of kinetic energy assuming the electrons start from rest. (b) Calculate the speed of the electrons using the classical form of kinetic energy. (c) Is the difference in speed significant in the design of this set in your opinion?
16. As noted in Section 2.2, the quantity $E - p^2c^2$ is an *invariant* in relativity theory. This means that the quantity $E^2 - p^2c^2$ has the same value in all inertial frames even though E and p have different values in different frames. Show this explicitly by considering the following case. A particle of mass m is moving in the $+x$ direction with speed u and has momentum p and energy E in the frame S. (a) If S′ is moving at speed v in the standard way, find the momentum p' and energy E' observed in S′. (*Hint:* Use the Lorentz velocity transformation to find p' and E'. Does $E = E'$ and $p = p'$? (b) Show that $E^2 - p^2c^2$ is equal to $E'^2 - p'^2c^2$.

2.3 Mass as a Measure of Energy

17. A radium isotope decays to a radon isotope, ^{222}Rn, by emitting an α particle (a helium nucleus) according to the decay scheme $^{226}\text{Ra} \to {}^{222}\text{Rn} + {}^{4}\text{He}$. The masses of the atoms are 226.0254 (Ra), 222.0175 (Rn), and 4.0026 (He). How much energy is released as the result of this decay?
18. Consider the decay ${}^{55}_{24}\text{Cr} \to {}^{55}_{25}\text{Mn} + \text{e}^-$, where e^- is an electron. The ^{55}Cr nucleus has a mass of 54.9279 u, and the ^{55}Mn nucleus has a mass of 54.9244 u. (a) Calculate the mass difference in MeV. (b) What is the maximum kinetic energy of the emitted electron?
19. Calculate the binding energy in MeV per nucleon in the isotope ${}^{12}_{6}$C. Note that the mass of this isotope is exactly 12 u, and the masses of the proton and neutron are 1.007276 u and 1.008665 u, respectively.
20. The free neutron is known to decay into a proton, an electron, and an antineutrino $\overline{v}$ (of negligible rest mass) according to

$$\text{n} \longrightarrow \text{p} + \text{e}^- + \overline{v}$$

This is called *beta decay* and will be discussed further in Chapter 13. The decay products are measured to have a total kinetic energy of 0.781 MeV ± 0.005 MeV. Show that this observation is consistent with the excess energy predicted by the Einstein mass–energy relationship.

2.4 Conservation of Relativistic Momentum and Energy

21. An electron having kinetic energy $K = 1.000$ MeV makes a head-on collision with a positron at rest. (A positron is an antimatter particle that has the same mass as the electron but opposite charge.) In the collision the two particles annihilate each other and are replaced by two γ rays of equal energy, each traveling at equal angles θ with the electron's direction of motion. (Gamma rays are massless particles of elec-

tromagnetic radiation having energy $E = pc$.) Find the energy E, momentum p, and angle of emission θ of the γ rays.

22. The K^0 meson is an uncharged member of the particle "zoo" that decays into two charged pions according to $K^0 \rightarrow \pi^+ + \pi^-$. The pions have opposite charges, as indicated, and the same mass, $m_\pi = 140\ \text{MeV}/c^2$. Suppose that a K^0 at rest decays into two pions in a bubble chamber in which a magnetic field of 2.0 T is present (see Fig. P2.22). If the radius of curvature of the pions is 34.4 cm, find (a) the momenta and speeds of the pions and (b) the mass of the K^0 meson.
23. An unstable particle having a mass of 3.34×10^{-27} kg is initially at rest. The particle decays into two fragments that fly off with velocities of $0.987c$ and $-0.868c$. Find the rest masses of the fragments.

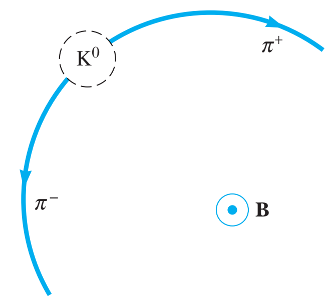

Figure P2.22 A sketch of the tracks made by the π^+ and π^- in the decay of the K^0 meson at rest. The pion motion is perpendicular to **B**. (**B** is directed out of the page.)

ADDITIONAL PROBLEMS

24. As measured by observers in a reference frame S, a particle having charge q moves with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ and an electric field $\mathbf{E}$. The resulting force on the particle is then measured to be $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. Another observer moves along with the charged particle and measures its charge to be q also but measures the electric field to be $\mathbf{E}'$. If both observers are to measure the same force, $\mathbf{F}$, show that $\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}$.
25. *Classical deflection of light by the Sun* Estimate the deflection of starlight grazing the surface of the Sun. Assume that light consists of particles of mass m traveling with velocity c and that the deflection is small. (a) Use $\Delta p_x = \int_{-\infty}^{+\infty} F_x\, dt$ to show that the angle of deflection θ is given by $\theta \cong \dfrac{2GM_s}{bc^2}$ where Δp_x is the total change in momentum of a light particle grazing the Sun. See Figures P2.25a and b. (b) For $b = R_s$, show that $\theta = 4.2 \times 10^{-6}$ rad.
26. An object having mass of 900 kg and traveling at a speed of $0.850c$ collides with a stationary object having mass 1400 kg. The two objects stick together. Find (a) the speed and (b) the mass of the composite object.
27. Imagine that the entire Sun collapses to a sphere of radius R_g such that the work required to remove a small mass m from the surface would be equal to its rest energy mc^2. This radius is called the *gravitational radius* for the Sun. Find R_g. (It is believed that the ultimate fate of very massive stars is to collapse beyond their gravitational radii into black holes.)
28. A rechargeable AA battery with a mass of 25.0 g can supply a power of 1.20 W for 50.0 min. (a) What is the difference in mass between a charged and an uncharged battery? (b) What fraction of the total mass is this mass difference?

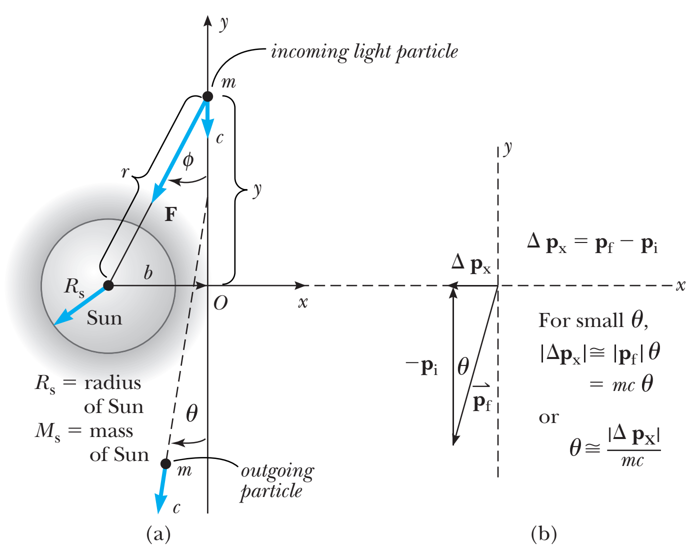

Figure P2.25 The classical deflection of starlight grazing the sun.

29. An object disintegrates into two fragments. One of the fragments has mass $1.00\ \text{MeV}/c^2$ and momentum $1.75\ \text{MeV}/c$ in the positive x direction. The other fragment has mass $1.50\ \text{MeV}/c^2$ and momentum $2.005\ \text{MeV}/c$ in the positive y direction. Find (a) the mass and (b) the speed of the original object.
30. The creation and study of new elementary particles is an important part of contemporary physics. Especially

interesting is the discovery of a very massive particle. To create a particle of mass M requires an energy Mc^2. With enough energy, an exotic particle can be created by allowing a fast-moving particle of ordinary matter, such as a proton, to collide with a similar target particle. Let us consider a perfectly inelastic collision between two protons: An incident proton with mass m, kinetic energy K, and momentum magnitude p joins with an originally stationary target proton to form a single product particle of mass M. You might think that the creation of a new product particle, 9 times more massive than in a previous experiment, would require just 9 times more energy for the incident proton. Unfortunately, not all of the kinetic energy of the incoming proton is available to create the product particle, since conservation of momentum requires that after the collision the system as a whole still must have some kinetic energy. Only a fraction of the energy of the incident particle is thus available to create a new particle. You will determine how the energy available for particle creation depends on the energy of the moving proton. Show that the energy available to create a product particle is given by

$$Mc^2 = 2mc^2\sqrt{1 + \frac{K}{2mc^2}}$$

From this result, when the kinetic energy K of the incident proton is large compared to its rest energy mc^2, we see that M approaches $(2mK)^{1/2}/c$. Thus if the energy of the incoming proton is increased by a factor of 9, the mass you can create increases only by a factor of 3. This disappointing result is the main reason that most modern accelerators, such as those at CERN (in Europe), at Fermilab (near Chicago), at SLAC (at Stanford), and at DESY (in Germany), use *colliding beams*. Here the total momentum of a pair of interacting particles can be zero. The center of mass can be at rest after the collision, so in principle all of the initial kinetic energy can be used for particle creation, according to

$$Mc^2 = 2mc^2 + K = 2mc^2\left(1 + \frac{K}{2mc^2}\right)$$

where K is the total kinetic energy of two identical colliding particles. Here, if $K >> mc^2$, we have M directly proportional to K, as we would desire. These machines are difficult to build and to operate, but they open new vistas in physics.

31. A particle of mass m moving along the x-axis with a velocity component $+u$ collides head-on and sticks to a particle of mass $m/3$ moving along the x-axis with the velocity component $-u$. What is the mass M of the resulting particle?
32. Compact high-power lasers can produce a 2.00-J light pulse of duration 100 fs focused to a spot 1 μm in diameter. (See Mourou and Umstader, "Extreme Light," *Scientific American*, May 2002, p. 81.) The electric field in the light accelerates electrons in the target material to near the speed of light. (a) What is the average power of the laser during the pulse? (b) How many electrons can be accelerated to $0.9999c$ if 0.0100% of the pulse energy is converted into energy of electron motion?
33. Energy reaches the upper atmosphere of the Earth from the Sun at the rate of 1.79×10^{17} W. If all of this energy were absorbed by the Earth and not re-emitted, how much would the mass of the Earth increase in 1.00 yr?