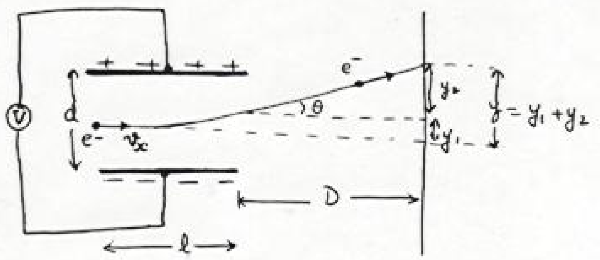
# Chapter 3

PS #4.




$$y = y_1 + y_2$$

$$y_1 = \frac{1}{2} a_y t_1^2 = \frac{1}{2}\left(\frac{F}{m_e}\right)\left(\frac{\ell}{v_x}\right)^2 \quad \left[\text{because } a_y = \frac{F}{m_e} \text{ and } t_1 = \frac{\ell}{v_x}\right]$$

$$y_1 = \frac{1}{2}\frac{Ee}{m_e}\frac{\ell^2}{v_x^2} = \frac{1}{2}\frac{V}{d}\left(\frac{e}{m_e}\right)\frac{\ell^2}{v_x^2} \quad \left[\text{because } E = \frac{V}{d}\right] \quad \text{—(1)}$$

$$y_2 = D\tan\theta = D\frac{v_y}{v_x} = D\frac{a_y t_1}{v_x} = D\frac{Ee}{m_e}\frac{\ell}{v_x^2}$$

$$y_2 = D\frac{V}{d}\left(\frac{e}{m_e}\right)\frac{\ell}{v_x^2} \quad \text{—(2)}$$

add (1) + (2) ⇒

$$\Rightarrow y_1 + y_2 = y = \left(\frac{e}{m_e}\right)\left[\frac{V}{2d}\frac{\ell^2}{v_x^2} + D\frac{V}{d}\frac{\ell}{v_x^2}\right] = \left(\frac{e}{m_e}\right)\frac{V\ell}{dv_x^2}\left(\frac{\ell}{2} + D\right)$$

$$\Rightarrow \boxed{\frac{e}{m_e} = \frac{y\,d\,v_x^2}{V\ell\left(\frac{\ell}{2} + D\right)}}$$

Pb # 7

a. radius of the drop of oil = 0.000276 cm

$\rho$: density of oil = 956.1 Kg/m$^3$

t = average time of fall (field $\vec{E}$ off) = 11.894 sec

$\bar{l}$: rise (field on) and fall (field off) distance = 10.21 mm

d: plate separation = 16.00 mm

V: Potential difference between the plates = 5085 V

∴ mass of the drop $m = \rho V = \rho \frac{4}{3}\pi a^3 = 8.42 \times 10^{-14}$ Kg

From textbook $q = \frac{mg}{E}\left(\frac{v+v'}{v}\right) = \underbrace{\frac{mgd}{V}}_{= 25.96 \times 10^{-19}\,C}\left(\frac{v+v'}{v}\right)$

$v$: speed of the drop when field off.

$v'$: speed " " " " on.

$v = \frac{\bar{l}}{t} = 8.58 \times 10^{-4}$ m/s

$v_1' = \frac{10.21 \times 10^{-3}}{80.708} = 1.27 \times 10^{-4}$ m/s $\Rightarrow q_1 = 25.96 \times 10^{-19}(1.148)$

$= 29.8 \times 10^{-19}$ C

$v_2' = \frac{10.21 \times 10^{-3}}{22.386} = 4.56 \times 10^{-4}$ m/s $\Rightarrow q_2 = 39.76 \times 10^{-19}$ C

$v_3' = \frac{10.21 \times 10^{-3}}{140.566} = 0.726 \times 10^{-4}$ m/s $\Rightarrow q_3 = 28.16 \times 10^{-19}$ C

$v_4' = \frac{10.21 \times 10^{-3}}{79.6} = 1.28 \times 10^{-4}$ m/s $\Rightarrow q_4 = 29.76 \times 10^{-19}$ C

$v_5' = 2.94 \times 10^{-4}$ m/s $\Rightarrow q_5 = 34.77 \times 10^{-19}$ C

$v_2' = 3.49 \times 10^{-4}$ m/s $\Rightarrow q_6 = 36.42 \times 10^{-19}$ C

$\Delta q$ (charge difference) $\qquad e = \frac{\Delta q}{n}$

$\Delta q_{21} = q_2 - q_1 = 9.98 \times 10^{-19} C \qquad e_1 = \frac{\Delta q_{21}}{6} = 1.66 \times 10^{-19} C$

$\Delta q_{13} = q_1 - q_3 = 1.64 \times 10^{-19} C \qquad e_2 = \frac{\Delta q_{13}}{1} = 1.64 \times 10^{-19} C$

$\Delta q_{14} = q_1 - q_4 \simeq 0 \qquad 0$

$\Delta q_{51} = q_5 - q_1 = 4.97 \times 10^{-19} C \qquad e_3 = \frac{\Delta q_{51}}{3} = 1.66 \times 10^{-19} C$

$\Delta q_{61} = q_6 - q_1 = 6.62 \times 10^{-19} C \qquad e_4 = \frac{\Delta q_{61}}{4} = 1.65 \times 10^{-19} C$

$\Delta q_{23} = q_2 - q_3 = 11.6 \times 10^{-19} C \qquad e_5 = \frac{\Delta q_{23}}{7} = 1.66 \times 10^{-19} C$

$\Delta q_{24} = q_2 - q_4 = 10 \times 10^{-19} C \qquad e_6 = \frac{\Delta q_{24}}{6} = 1.67 \times 10^{-19} C$

$q_{25} = q_2 - q_5 = 4.99 \times 10^{-19} C \qquad e_7 = \frac{\Delta q_{25}}{3} = 1.66 \times 10^{-19} C$

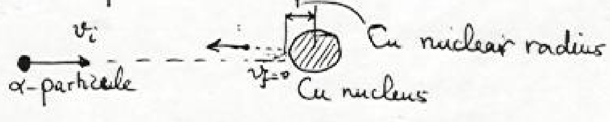
$\Delta q_{26} = q_2 - q_6 = 3.34 \times 10^{-19} C \qquad e_8 = \frac{\Delta q_{26}}{2} = 1.67 \times 10^{-19} C$

$\vdots \qquad \vdots$

$$\boxed{e_{average} = 1.659 \times 10^{-19} C}$$

Pb # 9

Rutherford Scattering formula will no longer hold when $\alpha$ particles begin to penetrate or touch the nucleus.




Conservation of energy $\quad K_i + U_i = K_f + U_f$

initially the $\alpha$-particle is far from the nucleus $\Rightarrow U_i = 0$

finally the $\alpha$-particle touches the surface of the nucleus and stops there momentarily $\Rightarrow K_f = 0$

$$\Rightarrow \quad K_i = U_f$$

The $\alpha$-particle has a charge $2e$ and the nucleus has a charge $Ze$.

$$K_\alpha = \frac{k(Ze)(2e)}{r}$$

$$\Rightarrow \quad r = \frac{k(Ze)(2e)}{K} = \frac{9\times10^{9} \times 29 \times 2 \times (1.6\times10^{-19})^2}{13.9\times1.6\times10^{-19}\times1\times10^{6}}$$

$$\boxed{r = 6.0\times10^{-15}\ \text{m}}$$

Pb# 14

a) For hydrogen atom $r_n = a_0 n^2 \quad n = 1, 2, 3, 4, \ldots$

$a_0 = 0.0529$ nm (Bohr radius)

First orbit $r_1 = a_0 = \boxed{0.0529\ \text{nm}}$

$2^{nd}$ orbit $r_2 = 4a_0 = \boxed{0.2116\ \text{nm}}$

$3^{rd}$ orbit $r_3 = 9a_0 = \boxed{0.4761\ \text{nm}}$

b) $$\frac{ke^2}{2r} = \frac{1}{2}m_e v^2 \Rightarrow v_n = \sqrt{\frac{ke^2}{m_e r_n}} = \frac{1}{n}\sqrt{\frac{ke^2}{m_e a_0}} \quad n=1,2,\ldots$$

First orbit $v_1 = \sqrt{\frac{ke^2}{m_e a_0}} = \boxed{2.19\times10^{6}\ \text{m/s}}$

$2^{nd}$ orbit $v_2 = \frac{v_1}{n} = \frac{v_1}{2} = \boxed{1.09\times10^{6}\ \text{m/s}}$

$3^{rd}$ orbit $v_3 = \frac{v_1}{3} = \boxed{0.73\times10^{6}\ \text{m/s}}$

c) $v_1 = 7.3 \times 10^{-3} c \ll c$

$v_2 = 3.6 \times 10^{-3} c \ll c$

$v_3 = 2.4 \times 10^{-3} c \ll c$

Therefore no relativistic correction is needed.

Pb # 16.

$Li^{2+}$ $Z = 3$

$Li^{2+}$ has only <u>one</u> electron. It is a hydrogen like ion.

$$E_n = -\frac{13.6}{n^2}(Z^2) = -\frac{13.6}{n^2}(9) = -\frac{122.4}{n^2} \text{ (eV)}$$

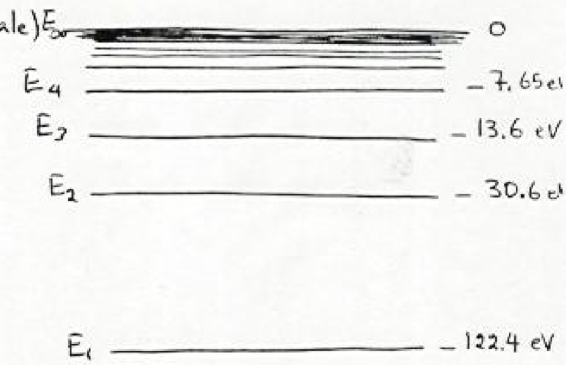
$E_1 = -122.4 \text{ eV}$ ground state

$E_2 = -\frac{122.4}{4} = -30.6 \text{ eV}$ first excited state

$E_3 = -\frac{122.4}{9} = -13.6 \text{ eV}$ 2nd excited state

$E_4 = -\frac{122.4}{16} = -7.65 \text{ eV}$ 3rd excited state

Energy diagram (not to scale)



Pb # 26.

For Lyman series $n_f = 1$ $n_i = 2, 3, 4, \ldots$

Longest wavelength corresponds to the smallest $\Delta E$

$\Rightarrow$ it is the transition $n_f = 1$ $n_i = 2$

In general $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$$\Rightarrow \frac{1}{\lambda_{max}} = R\left(\frac{1}{1} - \frac{1}{4}\right) = \frac{3R}{4} \Rightarrow \lambda_{max} = \frac{4}{3R}$$

$$\lambda_{max} = \frac{4}{3 \times 1.097 \times 10^7} = \boxed{121.51\ nm}$$

Shortest wavelength corresponds to the largest $\Delta E$

$\Rightarrow$ it is the transition with $n_f = 1$ $n_i = \infty$

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{1} - \frac{1}{\infty}\right) = R \Rightarrow \lambda_{min} = \frac{1}{R}$$

$$\lambda_{min} = \frac{1}{1.097 \times 10^7} = \boxed{91.13\ nm}$$

Both of these wavelengths are in the U.V. region

Pb # 36.

a) $f_n = \frac{1}{T_n} = \frac{v_n}{2\pi r_n}$ $v_n = \sqrt{\frac{k e^2}{m_e r_n}}$ and $r_n = a_0 n^2$

$$\Rightarrow f_n = \frac{1}{2\pi}\sqrt{\frac{k e^2}{m_e r_n^3}}$$

$n = 100$ $r_{100} = 0.0529 \times 10^{-9} (100)^2 = 0.0529 \times 10^{-5}$ m

$f_{100} = 6.58 \times 10^9$ HZ

$n = 1000 \quad r_{1000} = 0.0529 \times 10^{-9} \times (1000)^2 = 0.0529 \times 10^{-3}\, m$

$$f_{1000} = 6.58 \times 10^{6} \text{ HZ}$$

$n = 10\,000 \quad r_{10000} = 0.0529 \times 10^{-9} (10000)^2 = 0.0529 \times 10^{-1}\, m$

$$f_{10000} = 6.58 \times 10^{3} \text{ HZ}$$

b) $n_i = n \longrightarrow n_f = n-1$

$$f_n = \frac{\Delta E}{h} = \frac{13.6 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] \quad (\text{HZ})$$

$$= 3.285 \times 10^{15} \left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)$$

$n = 100 \quad f_{100} = 3.285 \times 10^{15} \left(\frac{1}{99^2} - \frac{1}{100^2}\right) = 6.67 \times 10^{9} \text{ HZ}$

$n = 1000 \quad f_{1000} = 3.285 \times 10^{15} \left(\frac{1}{999^2} - \frac{1}{1000^2}\right) = 6.58 \times 10^{6} \text{ HZ}$

$n = 10000 \quad f_{10000} = 3.285 \times 10^{15} \left(\frac{1}{9999^2} - \frac{1}{10\,000^2}\right) = 6.57 \times 10^{3} \text{ HZ}$

It is clear from this example that when n becomes large the classical frequency of revolution of the electron calculated in (a) is equal to the quantum frequency calculated in (b) $\Rightarrow$ the Bohr correpondence principale is valid.