KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

PHYSICS DEPARTMENT

PHYS212

FIRST MAJOR EXAM

Saturday 09 MARCH 2013

INSTRUCTOR: DR A MEKKI

EXAM TIME: 90 MINUTES

NAME: Key ID#

WRITE YOUR SOLUTIONS CLEARLY AND NEATLY. SHOW THE DETAILS OF YOUR SOLUTION. YOUR FINAL ANSWER SHOULD BE CIRCLED. NO CREDITS WILL BE GIVEN FOR ANSWERS WITHOUT A PROOF.

PROBLEM	GRADE/10
1	
2	
3	
4	
5	
6	
7	
TOTAL/70	
TOTAL/20	

Question 1 (10 points)

(a) A Physics student claims in court that the reason he crossed the red light (λ = 700 nm) was that due to his motion, the red light was Doppler shifted to green (λ = 500 nm). How fast was he going? (5 points).

$\lambda_s = 700 \text{ nm} \qquad \lambda_o = 500 \text{ nm}$

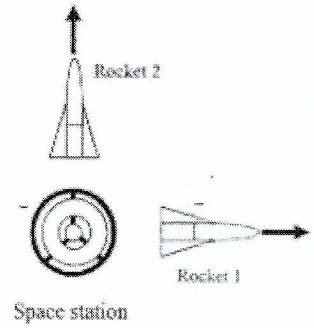
Since the observer is approaching the source $f_o = \dfrac{\sqrt{1+v/c}}{\sqrt{1-v/c}} f_s$

$$\left(\frac{f_o}{f_s}\right)^2 = \frac{1+v/c}{1-v/c} \Rightarrow \left(\frac{f_o}{f_s}\right)^2 (1-v/c) = 1 + v/c$$

$$\Rightarrow \frac{v}{c}\left[1+\left(\frac{f_o}{f_s}\right)^2\right] = \left(\frac{f_o}{f_s}\right)^2 - 1 \Rightarrow v/c = \frac{(f_o/f_s)^2 - 1}{(f_o/f_s)^2 + 1} = \frac{(\lambda_s/\lambda_o)^2 - 1}{(\lambda_s/\lambda_o)^2 + 1}$$

$$\frac{v}{c} = \frac{\left(\frac{7}{5}\right)^2 - 1}{\left(\frac{7}{5}\right)^2 + 1} = 0.324 \Rightarrow \boxed{v = 0.324c}$$

(b) Two rockets are leaving their space station along perpendicular paths, as seen measured by an observer on a stationary space station. Rocket 1 moves at 0.70c and rocket 2 moves at 0.90c, both relative to the space station. What is the velocity of rocket 2 as observed by rocket 1? (5 points)



rocket 1 $\quad u_x = 0.7c \quad u_y = 0$

rocket 2 $\quad u_x = 0 \quad u_y = 0.9c$

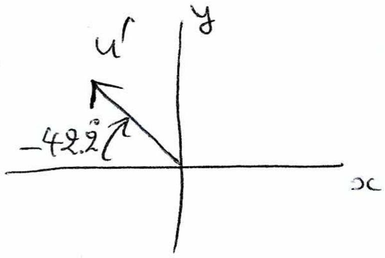
$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{\cancel{0} - 0.7c}{1 - 0}$$

$$= -0.7c$$

$$u_y' = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)} = \frac{0.9c}{1-0}\sqrt{1-(0.7)^2} = 0.64c$$

$$u' = \sqrt{u_x'^2 + u_y'^2} = \boxed{0.95c}$$

$$\theta = \tan^{-1}\left(\frac{-0.64}{0.7}\right) = -42.4^\circ$$



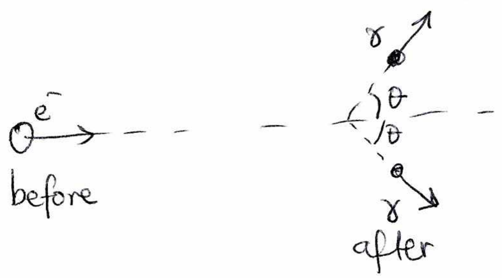
Question 2 (10 points)

An electron having kinetic energy K = 5.0 MeV makes a head-on collision with a positron at rest (A positron is an antimatter particle that has the same mass as the electron but opposite charge). In the collision the two particles annihilate each other and are replaced by two γ rays of equal energy, each travelling at equal angles θ with the electron's direction of motion. Suppose the electron was traveling in the positive x-direction before collision. (γ Rays are massless particles). Find the

(a) The energy E_γ of the γ rays in MeV. (3 points)

Conservation of mass-energy : $K_e + 2 m_e c^2 = 2 E_\gamma$

$$E_\gamma = \frac{K_e + 2 m_e c^2}{2} = \frac{5\text{ MeV} + 2 \times 0.511\text{ MeV}}{2} = \boxed{3.01\text{ MeV}}$$



(b) The momentum p_γ of the γ rays in MeV/c. (4 points)

Conservation of momentum

x-axis : $P_{e^-} = 2 P_\gamma \cos\theta$

$$E_\gamma^2 = P_\gamma^2 c^2 \Rightarrow P_\gamma = \frac{E_\gamma}{c} = 3.01 \frac{\text{MeV}}{c}$$

(c) The angle θ of the γ rays. (3 points)

$$\cos\theta = \frac{P_{e^-}}{2P_\gamma}$$

$$E_e^2 = P_e^2 c^2 + \cancel{(m_e c^2)^2} = (K_e + m_e c^2)^2 = K_e^2 + 2K_e m_e c^2 + \cancel{(m_e c^2)^2}$$

$$P_e = \frac{1}{c}\sqrt{K_e^2 + 2 K_e m_e c^2} = 5.48 \frac{\text{MeV}}{c}$$

$$\cos\theta = \frac{5.48}{2 \times 3.01} = 0.91 \Rightarrow \boxed{\theta = 24.5^\circ}$$

Question 3 (10 points)

A radium isotope, ^{226}Ra, decays into a radon isotope, ^{222}Rn, by emitting an α particle (helium nucleus) ^{4}He. The masses are 226.0254u for Ra, 222.0175u for Rn, and 4.0026u for He.

(a) Write the equation for this decay. (2 point)

$$^{226}Ra \longrightarrow {}^{222}Rn + {}^{4}He$$

(b) How much energy is released at a result of this decay? (8 points)

$$\Delta M = M_{Ra} - (M_{Rn} - M_{He})$$

$$= 226.0254\,u - (222.0175 + 4.0026)\,u$$

$$= 5.30 \times 10^{-3}\,u = 8.8 \times 10^{-30}\,Kg$$

$$E = \Delta M c^2 = 8.8 \times 10^{-30} \times (3 \times 10^{8})^2$$

$$= 7.9 \times 10^{-13}\,J = \boxed{4.9\,MeV}$$

Question 4 (10 points)

(a) Explain briefly what is the significance of Planck's law, i.e., to what aspect of physics does it apply, and what is the most important concept introduced by Planck to derive this law? (4 points)

This law was derived by Max Planck in 1900 to fit the blackbody radiation curve (energy density vs. wavelength). Planck assumed the resonators in the walls of the blackbody have discrete energies nhf and energy is radiated as $\Delta E = hf$.

(b) Show that from Planck's law, we can recover Wien's exponential law. (2 points)

Wien's law apply at low wavelengths or high frequencies

$$e^{hf/kT} \gg 1 \Rightarrow e^{hf/kT} - 1 = e^{hf/kT}$$

$$\Rightarrow u(f,T) = \frac{8\pi h}{c^3} f^3 e^{-hf/kT} = A f^3 e^{-\beta f/T}$$

(c) Show that from Plank's law we can recover Rayleigh-Jeans law.
(Note that for small values of x, $e^x = 1 + x$) (2 points)

Rayleigh-Jeans law apply at high wavelengths or low frequencies

$$e^{hf/kT} = \frac{hf}{kT} + 1$$

$$u(f,T) = \frac{8\pi f^2}{c^3} \frac{hf}{hf/kT} = \frac{8\pi kT}{c^3} f^2$$

(d) Explain briefly what are the problems with Wien's law and Rayleigh-Jeans law. (2 points)

Wien's law does not fit the u-V experimatal data

Rayleigh-Jeans law blow up (goes to ∞) for small λ or large f.

Question 5 (10 points)

Compton used in his experiment incident photons of wavelength 0.711 Å aimed at a block of carbon.

(a) What is the energy of the incident photon? (3 points)

$$E_p = hf = \frac{hc}{\lambda} = \frac{1240 \text{ eV}.\cancel{nm}}{0.0711 \cancel{nm}} = \boxed{17.4 \text{ keV}}$$

(b) What is the energy of the scattered photon at an angle of 180°? (3 points)

$$\Delta\lambda = \lambda_c (1 - \cos\theta) \qquad \theta = 180^\circ \qquad \cos\theta = -1$$

$$\Rightarrow \Delta\lambda = 2\lambda_c \quad \Rightarrow \lambda' = \lambda + 2\Delta\lambda_c = 0.7596 \text{ Å}$$

$$E_p' = \frac{hc}{\lambda'} = \frac{1240 \text{ eV}.\cancel{nm}}{0.07596 \cancel{nm}} = \boxed{16.3 \text{ keV}}$$

(c) What is the kinetic energy of the recoil electrons in the case of scattered photons at an angle of 180°? (3 points)

Conservation of energy $E_p = E_p' + K_e$

$$\Rightarrow K_e = E_p - E_p' = 17.4 - 16.3 = \boxed{1.1 \text{ keV}}$$

(d) The so-called free electrons in carbon are actually electrons with a binding energy of about 4 eV. Why may this binding energy be ignored in this case? (1 point)

The energy of the incident photon is 17.4 keV

Therefore, the electron binding energy of 4 eV is negligible in comparison with the incident X-ray energy

$$\text{ratio} = \frac{4}{17.4} \times 10^{-3} = 0.23 \times 10^{-3}$$

Question 6 (10 points)

For the triply ionized Beryllium ion (Z = 4) calculate

(a) The energy of the electron in the third excited state. (2 points)

$$E_n = -\frac{13.6}{n^2} Z^2 \ (eV)$$

$Z=4$ and $n=4$ $\quad E_4 = -\frac{13.6}{(4)^2}(4)^2 = \boxed{-13.6\ eV}$

(b) The radius of that orbit. (2 points)

$$r_n = \frac{a_0}{Z} n^2 \qquad r_4 = \frac{a_0}{4}(4)^2 = 4a_0 = \boxed{0.212\,nm}$$

(c) The speed of the electron in that orbit. (2 points)

$$m v r = n\hbar \qquad v = \frac{n\hbar}{mr} \Rightarrow v_n = \frac{n\hbar}{m \cdot r_n}$$

$$v_4 = \frac{4 \times 1.05 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.212 \times 10^{-9}} = 2.18 \times 10^{6}\ m/s.$$

$$\boxed{v_4 = 2.18 \times 10^{6}\ m/s}$$

(d) The wavelength of light emitted when the electron "jumps" from the third excited state to the ground state. (2 points) $\quad n_i = 4 \rightarrow n_f = 1 \qquad Z = 4$

$$\Delta E = \frac{hc}{\lambda} = 13.6 Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = 13.6 \times (4)^2 \left(\frac{1}{1} - \frac{1}{16}\right)$$

$$\frac{hc}{\lambda} = 204\ eV \Rightarrow \lambda = \frac{hc}{204\,eV} = \frac{1240\ \cancel{eV} \cdot nm}{204\ \cancel{eV}}$$

$$\boxed{\lambda = 6.1\ nm}$$

(e) Is this wavelength in the visible spectrum? Explain. (2 points)

No, the visible spectrum is from 400nm to 700 nm.

Question 7 (10 points)

(a) Who discovered the charge to mass ratio of the electron? (0.5 point)

J. J. Thomson

(b) Who found the value of the electron charge to a high precision? (0.5 point)

R. Millikan

(c) Who discovered X-rays? (0.5 point)

W. Roentgen

(d) Who did a thorough experimental study of the photoelectric effect? (0.5 point)

P. Lenard

(e) Which experiment offered evidence of the wave-particle duality of light? (0.5 point)

The Compton effect

(f) Who was the first to introduce the concept of quantization of light? (0.5 point)

A. Einstein

(g) What is the significance of the Frank-Hertz experiment? (1 points)

It was a direct evidence of quantization of the energy levels of atoms.

(h) What is meant by the "Bohr correspondence principle"? (1 points)

As the quantum number $n \to \infty$ quantum physics = Classical physics

(i) What did Michelson-Morley experiment prove? (0.5 point)

The experiment proved the non-existance of the ether

(j) Who found the theory that explains the spectral lines of atoms? (0.5 point)

N. Bohr

(k) What is the physical meaning of the work function of a metal? (0.5 point)

It is the minimum energy with which an electron is bound in the metal.

(l) Who found experimentally Planck's constant to a high precision? (0.5 point)

R. Millikan

(m) Is the relativistic kinetic energy lost during an inelastic collision? Explain(1 point)

No, it is converted into mass.

(n) How do you calculate the convergence limit of spectral lines in the Balmer series? (1 point)

It is the wavelength for emission from $n_i = \infty$ to $n_f = 2$.

(o) What does the red shift observed in stars indicate? (1 point)

It indicates that stars are moving away from us and that the universe is expanding.

PHYS212- FORMULA SHEET – MAJOR 1
Term122

$$\Delta L = \frac{\Delta L'}{\gamma} \qquad \Delta t = \gamma \Delta t' \qquad x' = \gamma(x - vt) \qquad t' = \gamma(t - \frac{v}{c^2}x)$$

$$u_x' = \frac{u_x - v}{1 - (\frac{u_x v}{c^2})} \qquad u'_{y,z} = \frac{u_{y,z}}{\gamma\left[1 - (\frac{u_x v}{c^2})\right]} \qquad f_{obs} = \frac{\sqrt{1 \pm (v/c)}}{\sqrt{1 \mp (v/c)}} f_{source}$$

$$E^2 = p^2c^2 + (mc^2)^2 \qquad E = \gamma mc^2 = K + mc^2 \qquad p = \gamma mu$$

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \qquad eV_s = hf - \phi \qquad e = A\sigma T^4 \qquad E = nhf$$

$$\frac{e}{m} = \frac{V\theta}{B^2 l d} \qquad q = (\frac{mg}{E})(\frac{v + v'}{v}) \qquad m_e vr = n\hbar \qquad r_n = \frac{n^2 a_0}{Z}$$

$$E_n = \frac{-13.6\, Z^2}{n^2} \qquad \Delta n = \frac{k^2 Z^2 e^4 NnA}{4R^2 K^2 \sin^4(\frac{\phi}{2})} \qquad E_{res} = nhf \qquad u(\lambda, T) = \frac{8\pi hc}{\lambda^5 (e^{\frac{hc}{\lambda kT}} - 1)}$$

$$n\lambda = 2d\sin\theta \quad n = 1, 2, 3, \ldots \quad D = 6\pi a\eta v$$

Constants:

$e = 1.6 \times 10^{-19}\ C$ $\qquad m_e = 9.1 \times 10^{-31}\ kg$ $\qquad \hbar = 1.05 \times 10^{-34}\ J \cdot s$

$m_p = 1.67 \times 10^{-27}\ kg$ $\qquad c = 3 \times 10^8\ m/s$ $\qquad 1u = 1.66 \times 10^{-27}\ kg$

$k_B = 1.38 \times 10^{-23}\ J/K$ $\qquad k = 9 \times 10^9\ N \cdot m^2 C^2$ $\qquad m_e c^2 = 0.511\ MeV$

$m_p c^2 = 938\ MeV$ $\qquad hc = 12400\ eV \cdot A$ $\qquad \lambda_c = 0.00243\ nm$

$\sigma = 5.67 \times 10^{-8}\ W.m^{-2}\ K^{-4}$ $\qquad R = 1.0973 \times 10^7\ m^{-1}$ $\qquad a_o = 0.053 nm$