KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF PHYSCIS

PHYSICS 212 Modern Physics
TERM 042
EXAM #1

Date: Saturday 26 March 2004
Instructor: Dr. A. Mekki
Time of the exam: 90 minutes

Name: Key **Id#:**

SHOW THE DETAILS OF YOUR WORK

Question#	Maximum	Your grade
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
Total	45	
	15	

Q1. (5 points)

Reference frame S' moves with a velocity 0.6c in the positive x direction with respect to reference frame S. The clocks S and S' are synchronized to t = t' = 0 in the instant the coordinates origins of the two frames coincide. In frame S, two events, A and B, occur at x_A = 1.0 km, t_A = 3.0 μs and x_B = 1.5 km, t_B = 4.6 μs. Determine the time and position coordinates in the S' frame of events A and B.

In frame S (fixed frame) $x_A = 1.0$ Km $t_A = 3.0\ \mu s$

$x_B = 1.5$ Km $t_B = 4.6\ \mu s$

Using lorentz transformations we can find

$$x'_A = \gamma(x_A - v t_A) \qquad x'_B = \gamma(x_B - v t_B)$$

$$t'_A = \gamma\left(t_A - \frac{v}{c^2} x_A\right) \qquad t'_B = \gamma\left(t_B - \frac{v}{c^2} x_B\right)$$

where $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}} = \dfrac{1}{\sqrt{1-(0.6)^2}} = 1.25$

$$x'_A = 1.25(1000 - 0.6 \times 3\times10^8 \times 3\times10^{-6}) = 575\ \text{m}$$

$$= \boxed{0.575\ \text{Km}}$$

$$t'_A = 1.25\left(3\times10^{-6} - \frac{0.6}{3\times10^8} \times 1000\right) = \boxed{1.25\ \mu s}$$

$$x'_B = 1.25(1500 - 0.6\times 3\times10^8 \times 4.6\times10^{-6}) = 840\ \text{m}$$

$$= \boxed{0.840\ \text{Km}}$$

$$t'_B = 1.25\left(4.6\times10^{-6} - \frac{0.6}{3\times10^8} \times 1500\right) = \boxed{2.0\ \mu s}$$

Q2. (5 points)

Charged pions, π^+ and π^-, can be produced in high-energy collisions. Charged pions are unstable particles and decay with a half-life of 1.8×10^{-8} s (in the rest frame of the pion). Pions are produced in accelerators and emerge from the machine at a speed of 0.996c.

(a) What is the half life of the pion in the laboratory frame?

$$T' = \gamma T$$

where T is the proper time
T' is the time measured in the laboratory frame

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{1}{\sqrt{1 - (0.996)^2}} = 11.19$$

$$T' = 11.19 \times 1.8 \times 10^{-8} = \boxed{20.14 \times 10^{-8}\,s}$$

2

(b) How far do these particles travel in the laboratory frame (S) before half of them decay?

$$d' = v T' = 0.996 \times 3.10^{8} \times 20.14 \times 10^{-8}$$

$$= \boxed{60.2\ m}$$

1.5

(c) How far do the pions travel in a reference frame at rest with respect to them before half of them decay?

$$d = v T = 0.996 \times 3 \times 10^{-8} \times 1.8 \times 10^{-8}$$

1.5

$$= \boxed{5.38\ m}$$

Q3. (5 points)
A free neutron is known to decay into a proton, an electron, and an antineutrino $\bar{\nu}$ (of zero rest mass) according to

$$\mathbf{n \rightarrow p + e^- + \bar{\nu}}$$

This is called *beta decay*.
How much energy (in electronvolt) is released as a result of this decay?

The energy released is due to decrease in the mass so that $\Delta E = \Delta m c^2$

$$\Delta m = m_{before} - m_{after} = m_n - (m_p + m_{e^-} + m_{\bar{\nu}}^{\to 0})$$

$$= 1.09 \times 10^{-31} \text{ Kg}$$

$$\Rightarrow \Delta E = (1.09 \times 10^{-31}) \times (3 \times 10^8)^2 = 9.8 \times 10^{-14} \text{ J} = 6.1 \times 10^5 \text{ eV}$$

$$= \boxed{0.61 \text{ MeV}}$$

Q4. (5 points)
Planck blackbody radiation law is given by

$$u(f,T) = \frac{8\pi h f^3}{c^3}\left[\frac{1}{e^{\frac{hf}{k_B T}} - 1}\right]$$

Show that from the above general equation, we can recover Wien's exponential law and Rayleigh-Jeans law.
Note that for small values of x, $e^x = 1 + x$

- Wien's exponential law apply at low wavelengths or high frequencies

For high frequencies $e^{\frac{hf}{k_B T}} \gg 1 \Rightarrow e^{\frac{hf}{k_B T}} - 1 \simeq e^{\frac{hf}{k_B T}}$

$$\Rightarrow u(f,T) = \frac{8\pi h f^3}{c^3} \frac{1}{e^{hf/k_B T} - 1} = \frac{8\pi h f^3}{c^3} e^{-\frac{hf}{kT}}$$

$$\boxed{u(f,T) = A f^3 e^{-\beta \frac{f}{T}}}$$ where $A = \frac{8\pi h}{c^3}$ and $\beta = \frac{h}{k_B}$

- Rayleigh-Jeans law apply at high wavelength or low frequencies

For small frequencies $e^{\frac{hf}{k_B T}} \simeq \frac{hf}{k_B T} + 1$

$$\Rightarrow u(f,T) = \frac{8\pi h f^3}{c^3} \frac{1}{e^{\frac{hf}{k_B T}} - 1} = \frac{8\pi h f^3}{c^3} \frac{k_B T}{hf} = \frac{8\pi k_B}{c^3} f^2 T$$

$$\boxed{u(f,T) = \frac{8\pi k_B}{c^3} f^2 T}$$

Q7. (5 points)

(a) Find the frequencies of revolution of electrons in n = 1 and n = 2 Bohr orbits.

$$f_n = \frac{1}{T_n} = \frac{v_n}{2\pi r_n} \qquad \text{but} \quad m v_n r_n = n\hbar$$

$$\frac{v_n}{r_n} = \frac{n\hbar}{m r_n^2} = \frac{n\hbar}{m a_0^2 n^4}$$

$$\frac{v_n}{r_n} = \frac{\hbar}{m a_0^2 n^3}$$

$$\Rightarrow f_n = \frac{\hbar}{2\pi m a_0^2 n^3} = \left(\frac{\hbar}{2\pi m a_0^2}\right)\frac{1}{n^3}$$

$$\Rightarrow f_{1\,\text{orbital}} = 6.56 \times 10^{15}\ \text{Hz} \quad (n = 1)$$

$$f_{2\,\text{orbital}} = 8.20 \times 10^{14}\ \text{Hz} \quad (n = 2)$$

(b) What is the frequency of the photon emitted when an electron in an n = 2 orbit drops to an n = 1 orbit?

$$\Delta E = hf \Rightarrow f_{\text{photon}} = \frac{\Delta E}{h} = \frac{13.6}{h}\left(\frac{1}{1} - \frac{1}{4}\right) = 2.5 \times 10^{15}\ \text{Hz}$$

(c) Can you apply Bohr's correspondence principle in this case? Explain why?

No because since $f_{\text{photon}} \neq f_{\text{orbital}}$. Bohr's Correspondence principle applies only for $n \to \infty$.

(d) An electron typically spends about 10^{-8} s in an excited state before it drops to a lower level by emitting a photon. How many revolutions does an electron in an n = 2 orbit make in 1×10^{-8} s?

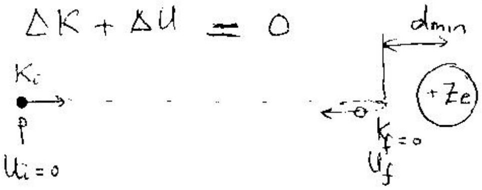
$$\text{For } n = 2 \quad f_2 = 8.2 \times 10^{14}\ \text{Hz} \Rightarrow T_2 = \frac{1}{f_2} = 1.2 \times 10^{-15}\ \text{sec}$$

$$\#\text{ of revolutions} \Rightarrow N = \frac{10^{-8}}{1.2 \times 10^{-15}} = 8.2 \times 10^{6}\ \text{revolutions}$$

Q8. (5 points)
Determine the distance of closest approach of 1.0 MeV protons incident on gold nuclei for which Z =79.

Use conservation of energy principle

$\Delta K + \Delta U = 0$



$$K - \frac{k(Ze)(e)}{d_{min}} \Rightarrow d_{min} = \frac{kZe^2}{K}$$

$$d_{min} = \frac{9\times10^{9} \times 79 \times (1.6\times10^{-19})^2}{1\times10^{6} \times (1.6\times10^{-19})} = \boxed{1.13\times10^{-13}\ m}$$

Q9. (5 points)

(a) Find the shortest wavelength present in the radiation from an x-ray machine whose accelerating potential is 50 kV.

The shortest wavelength correspond to the case where all the electron energy is converted to x-ray photon energy

$$\Rightarrow \quad eV = hf_{max} = \frac{hc}{\lambda_{min}} \Rightarrow \lambda_{min} = \frac{hc}{eV}$$

$$\lambda_{min} = \frac{12400\,eV\cdot Å}{50\times10^{3}\,eV} = \boxed{0.248\ Å}$$

(b) Does this wavelength depend on the target material? Explain.

No, since λ_{min} depends only on the electron accelerating voltage.