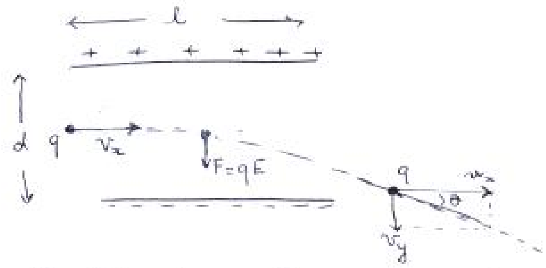
Chapter 3: The Particle nature of matter

#3.3.



$\theta = 0.2$ rad $\ell = 10$ cm $B = 4.57 \times 10^{-2}$ T

$V = 2000$ V $d = 2$ cm

a) $\tan\theta = \dfrac{v_y}{v_x}$ and $t = \dfrac{\ell}{v_x}$

$$v_y = a_y t = \frac{F}{m} t = \frac{qE}{m}\frac{\ell}{v_x} = \frac{q}{m}\frac{V}{d}\frac{\ell}{v_x}$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{q}{m}\frac{V}{d}\frac{\ell}{v_x^2}$$

but $qE = q v_x B$ (beam undeflected) velocity selector

$$\Rightarrow v_x = \frac{E}{B}$$

$$\Rightarrow \tan\theta = \frac{q}{m}\frac{V}{d}\frac{\ell B^2}{\left(\frac{V}{d}\right)^2} = \frac{q}{m}\frac{\ell B^2 d}{V}$$

if θ is small as is the case here $\tan\theta \simeq \theta$

$$\Rightarrow \frac{q}{m} = \frac{V\theta}{\ell d B^2} = \frac{2000 \times 0.2}{(0.1)(0.02)(4.57\times10^{-2})^2} = \boxed{9.58\times10^{7}\ C/Kg}$$

b) Since the deflection is toward the negative plate (see figure) the charge must be positive.

Note that for the proton $\frac{q}{m} = \frac{1.6\times10^{-19}}{1.67\times10^{-27}} = 9.58\times10^{7}\ C/Kg$.

This is a proton (or a beam of protons).

c) $v_x = \frac{E}{B} = \frac{V}{dB} = \frac{2000}{(0.02)(4.57\times10^{-2})} = \boxed{2.19\times10^{6}\ m/s}$

d) $\frac{v_x}{c} = \frac{2.19\times10^{6}}{3\times10^{8}} \simeq 0.01 \Rightarrow v_x = 0.01c$.

There is no need to use relativistic mechanics in this case.

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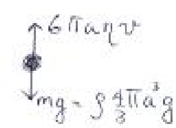
a) • The radius of the drop is $a = \sqrt{\frac{9\eta v}{2\rho g}}$ $\Leftarrow$



$v = \frac{x}{t} = 2.5\times10^{-4}\ m/s$

$\rho = 0.8\times10^{3}\ Kg/m^3$

$\Rightarrow \boxed{a = 1.62\times10^{-6}\ m}$

• The mass of the drop is

$\rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi a^3} \Rightarrow m = \rho\frac{4}{3}\pi a^3 = \boxed{1.42\times10^{-14}\ Kg}$

b)

$q_i = \frac{mg}{E}\left(\frac{v+v_i'}{v}\right) \quad i = 1, 2, 3, \ldots$

$q_i = \frac{mgd}{V}\left(\frac{v+v_i'}{v}\right)$

$v = 2.5\times10^{-4}\ m/s \quad v_1' = 1.1\times10^{-4}\ m/s \quad v_2' = 2.31\times10^{-4}\ m/s$

$v_3' = 1.67\times10^{-4}\ m/s \quad v_4' = 3.51\times10^{-4}\ m/s \quad v_5' = 5.31\times10^{-4}\ m/s$

$\Rightarrow$ $q_1 = 10 \times 10^{-19}$ C

$q_2 = 13.4 \times 10^{-19}$ C

$q_3 = 11.6 \times 10^{-19}$ C

$q_4 = 16.7 \times 10^{-19}$ C

$q_5 = 21.6 \times 10^{-19}$ C

c) However 1.5×10^{-19} C $< e < 2.0 \times 10^{-19}$ C

$e_1 = \frac{q_1}{6} = 1.67 \times 10^{-19}$ C

$e_2 = \frac{q_2}{8} = 1.68 \times 10^{-19}$ C

$e_3 = \frac{q_3}{7} = 1.66 \times 10^{-19}$ C

$e_4 = \frac{q_4}{10} = 1.67 \times 10^{-19}$ C

$e_5 = \frac{q_5}{13} = 1.66 \times 10^{-19}$ C

$$\boxed{\bar{e} = 1.67 \times 10^{-19}\ \text{C}}$$

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For emission lines we have the general formula

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

For Lyman series $n_f = 1$ $n_i = 2, 3, \ldots$

First line $n_i = 2$ $\Rightarrow \frac{1}{\lambda_1} = R_H \left(1 - \frac{1}{4} \right) \Rightarrow \lambda_1 = \frac{4}{3R} = 1251\,\text{Å}$ (UV)

Second line $n_i = 3$ $\Rightarrow \frac{1}{\lambda_2} = R_H \left(1 - \frac{1}{9} \right) \Rightarrow \lambda_2 = \frac{9}{8R} = 1025\,\text{Å}$ (UV)

Third line $n_i = 4$ $\Rightarrow \frac{1}{\lambda_3} = R_H \left(1 - \frac{1}{16} \right) \Rightarrow \lambda_3 = \frac{16}{15R} = 972\,\text{Å}$ (UV)

3.25

a) $$\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Here $n_i = 4$ and $n_f = 3$ $\Rightarrow \frac{1}{\lambda} = R_H \left(\frac{1}{9} - \frac{1}{16} \right)$

$$\Rightarrow \lambda = \frac{144}{7R}$$

The frequency of the photon is:

$$f = \frac{c}{\lambda} = \frac{c \times 7R}{144} = \boxed{1.6 \times 10^{14}\ \text{HZ}}$$

The frequency of revolution is: $f_n = \frac{1}{T_n} = \frac{v_n}{2\pi r_n} = \frac{1}{2\pi}\sqrt{\frac{k e^2}{m_e r_n^3}}$

$$\Rightarrow f_3 = \frac{1}{2\pi}\sqrt{2.35 \times 10^{30}} = 2.44 \times 10^{14}\ \text{Hz}$$

$$f_4 = \frac{1}{2\pi}\sqrt{4.2 \times 10^{29}} = 1.03 \times 10^{14}\ \text{Hz}$$

$$f_{photon} \simeq \frac{f_3 + f_4}{2} \quad !$$

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a) $f_n = \frac{1}{T_n} = \frac{v}{2\pi r_n}$

$\frac{1}{\cancel{2}} m_e v^2 = \frac{ke^2}{\cancel{2} r_n}$

$v = \sqrt{\frac{ke^2}{m_e r_n}}$

$$\boxed{f_{revol.} = \frac{1}{2\pi}\sqrt{\frac{k e^2}{m_e r_n^3}}}$$

$r_n = n^2 a_0 = 0.0529 \times 10^{-9}\, n^2$

$f_{100} = ?$ $r_{100} = 5.29 \times 10^{-7}$ m $f_{100} = 6.58 \times 10^{9}$ HZ

$f_{1000} = ?$ $r_{1000} = 5.29 \times 10^{-5}$ m $f_{1000} = 6.58 \times 10^{6}$ HZ

$f_{10000} = ?$ $r_{10000} = 5.29 \times 10^{-3}$ m $f_{1000} = 6.58 \times 10^{3}$ HZ

b) $n_i = n \longrightarrow n-1 = n_f$

$$\frac{1}{\lambda} = R\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right) = R$$

$$\Delta E = 13.6\ Z^2\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right) = hf$$

$$f = \frac{\Delta E}{h} = \frac{13.6\, Z^2}{h}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)$$

$$f_{photon} = \frac{13.6\,(eV)}{4.14 \times 10^{-15}\,(eV \cdot s)}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right) = 3.285 \times 10^{15}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)$$

$n = 100$ $f_{100} = 3.285 \times 10^{15}\left(\frac{1}{99^2} - \frac{1}{100^2}\right) = 6.67 \times 10^{9}$ HZ

$n = 1000$ $f_{1000} = 3.285 \times 10^{15}\left(\frac{1}{999^2} - \frac{1}{1000^2}\right) = 6.58 \times 10^{6}$ HZ

$n = 10000$ $f_{10\,000} = 3.285 \times 10^{15}\left(\frac{1}{9999^2} - \frac{1}{10\,000^2}\right) = 6.57 \times 10^{3}$ HZ

$\Rightarrow$ f_{photon} tends to $f_{revol.}$ for large n values!

the correspondance theorem is valid.