Chapter 4: Matter Waves

4.6

De broglie wavelength is given by $\lambda = \frac{h}{p}$

$$K = \frac{1}{2} m v^2 = \frac{1}{2} \frac{m^2 v^2}{m} = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK}$$

$$\Rightarrow \quad \lambda = \frac{h}{\sqrt{2mK}}$$

This is true for the non-relativistic case.

For the electron $\lambda = \dfrac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 50 \times 10^{3} \times 1.6 \times 10^{-19}}} = 5.5 \times 10^{-12} m = \boxed{5.5 \times 10^{-2} \mathring{A}}$

For the proton $\lambda = \dfrac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 50 \times 10^{3} \times 1.6 \times 10^{-19}}} = 1.3 \times 10^{-13} m = \boxed{1.3 \times 10^{-3} \mathring{A}}$

4.16

$$v_p = \left(\frac{2\pi}{\lambda}\right)^{1/2} \left(\frac{s}{\rho}\right)^{1/2} = k^{1/2} \left(\frac{s}{\rho}\right)^{1/2} \quad \text{since } k = \frac{2\pi}{\lambda}$$

We proved in the class that $v_g = v_p\big|_{k_0} + k \left.\dfrac{dv_p}{dk}\right|_{k_0}$

$$v_g = \left(\frac{s}{\rho}\right)^{1/2} k_0^{1/2} + k_0 \frac{1}{2} k_0^{-1/2} \left(\frac{s}{\rho}\right)^{1/2}$$

$$= \left(\frac{s}{\rho}\right)^{1/2} k_0^{1/2} + \frac{1}{2} \left(\frac{s}{\rho}\right)^{1/2} k_0^{1/2} = \frac{3}{2} \left(\frac{s}{\rho}\right)^{1/2} k_0^{1/2}$$

$$\boxed{v_g = \frac{3}{2} v_p\big|_{k_0}}$$

4.22

For this ball $p = mv = 0.05 \times 30 = 1.5 \ \text{Kg} \frac{m}{s}$

$$\frac{\Delta v}{v} = 0.001 \quad \Rightarrow \quad \Delta v = 0.001 \, v$$

$$\text{or} \quad m \Delta v = \Delta p = 0.001 \, mv = 0.001 \, p$$

$$\Rightarrow \Delta p = 0.001 \times 1.5 = 1.5 \times 10^{-3} \ \text{Kg} \cdot \frac{m}{s}$$

$$\Delta x \cdot \Delta p \geqslant \frac{\hbar}{2} \simeq \frac{\hbar}{2}$$

$$\Rightarrow \Delta x = \frac{\hbar}{2 \Delta p} = \frac{1.05 \times 10^{-34}}{2 \times 1.5 \times 10^{-3}} = \boxed{3.5 \times 10^{-32} \ m}$$

This result is too small to be detected by any available instrument. Therefore it can be neglected..

Heisenberg uncertainty principle is important for microscopic systems.

4.27

$$\Delta p \cdot \Delta x \geqslant \frac{\hbar}{2}$$

a) minimum uncertainty $\Rightarrow \Delta p . \Delta x = \frac{\hbar}{2}$

$$\Delta p = m \Delta v$$

$$\Rightarrow m \Delta v \Delta x = \frac{1}{2} \qquad (\text{since } h = 2\pi)$$

$$\Rightarrow \Delta v = \frac{1}{2 m \Delta x} = \frac{1}{2 \times 2 \times 1} = \boxed{0.25 \ m/s}$$

b) $\Delta x = \Delta v t = 0.25 \times 5 = 1.25m \Rightarrow \Delta x = 1 + 1.25 = \boxed{2.25m}$

4.36

a) $\Delta E = hf \Rightarrow f = \dfrac{\Delta E}{h} = \dfrac{1.8 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}}$

$\boxed{f = 4.4 \times 10^{14}\ \text{Hz}}$

b) $\lambda = \dfrac{c}{f} = \boxed{6.9 \times 10^{-7}\ \text{m}}$

c) $\Delta E \cdot \Delta t \geq \dfrac{\hbar}{2}$

$\Delta t = 2 \times 10^{-6}\ \text{s}$

$\Delta E = \dfrac{\hbar}{2\Delta t} = \dfrac{1.05 \times 10^{-34}}{2 \times 2 \times 10^{-6}} = \boxed{1.6 \times 10^{-8}\ \text{eV}}$