EXAMPLE 24.1 Flux Through a Cube

Consider a uniform electric field $\mathbf{E}$ oriented in the x direction. Find the net electric flux through the surface of a cube of edges ℓ oriented as shown in Figure 24.5.

Solution The net flux can be evaluated by summing up the fluxes through each face of the cube. First, note that the flux through *four* of the faces is zero, since $\mathbf{E}$ is perpendicular to $d\mathbf{A}$ on these faces. In particular, the orientation of $d\mathbf{A}$ is perpendicular to $\mathbf{E}$ for the two faces labeled ③ and ④ in Figure 24.5. Therefore, $\theta = 90°$, so that $\mathbf{E}\cdot d\mathbf{A} = E\,dA\cos 90° = 0$. The fluxes through the planes parallel to the yx plane are also zero for the same reason.

Now consider the faces labeled ① and ②. The net flux through these faces is given by

$$\Phi_e = \int_1 \mathbf{E}\cdot d\mathbf{A} + \int_2 \mathbf{E}\cdot d\mathbf{A}$$

For the face labeled ①, $\mathbf{E}$ is constant and inward while $d\mathbf{A}$ is outward ($\theta = 180°$), so that we find that the flux through this face is

$$\int_1 \mathbf{E}\cdot d\mathbf{A} = \int_1 E\,dA\cos 180° = -E\int_1 dA$$

$$= -EA = -E\ell^2$$

since the area of each face is $A = \ell^2$.

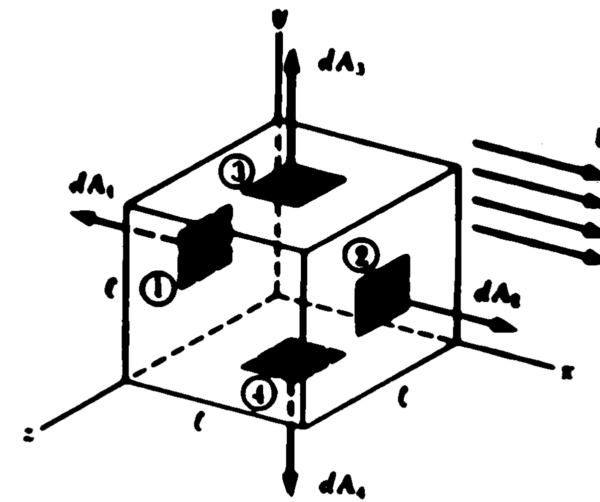


Figure 24.5 (Example 24.1) A hypothetical surface in the shape of a cube in a uniform electric field parallel to the x axis. The net flux through the surface is zero.

Likewise, for the face labeled ②, $\mathbf{E}$ is constant and outward and in the same direction as $d\mathbf{A}$ ($\theta = 0°$), so that the flux through this face is

$$\int_2 \mathbf{E}\cdot d\mathbf{A} = \int_2 E\,dA\cos 0° = E\int_2 dA = +EA = E\ell^2$$

Hence, the net flux over all faces is zero, since

$$\Phi_e = -E\ell^2 + E\ell^2 = 0$$

•2 In Fig. 22-29 the electric field lines on the left have twice the separation of those on the right. (a) If the magnitude of the field at A is 40 N/C, what is the magnitude of the force on a proton at A? (b) What is the magnitude of the field at B?

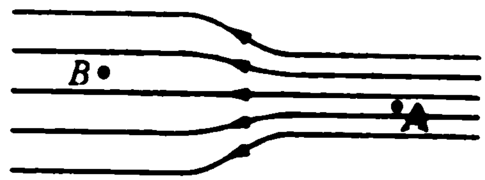


Fig. 22-29 Problem 2.

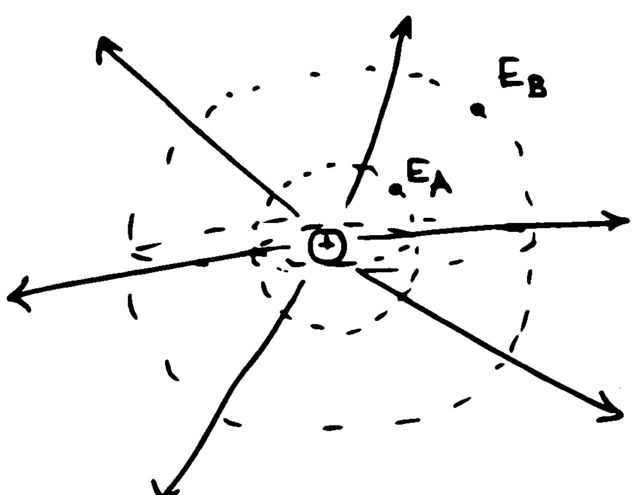
a) $|\vec{E}_A| = 40$ N/C

$$F_A = q_0 E_A = 1.6\times10^{-19} \times 40$$
$$= \boxed{6.4\times10^{-18}\ \text{N}}$$

direction is to the left

$$\vec{F}_A = -6.4\times10^{-18}\,\hat{i}\ (\text{N})$$

b) Since the separation at B is twice that at A $\Rightarrow$ $E_B = \frac{E_A}{2} = 20\ \frac{N}{C}$



electric field density is larger at A than at B.

••8 In Fig. 22-30, particle 1 of charge $q_1 = -5.00q$ and particle 2 of charge $q_2 = +2.00q$ are fixed to an x axis. (a) As a multiple of distance L, at what coordinate on the axis is the net electric field of the particles zero? (b) Sketch the net electric field lines.

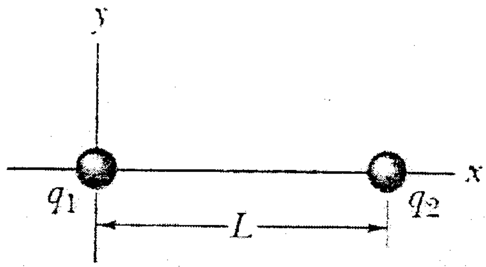
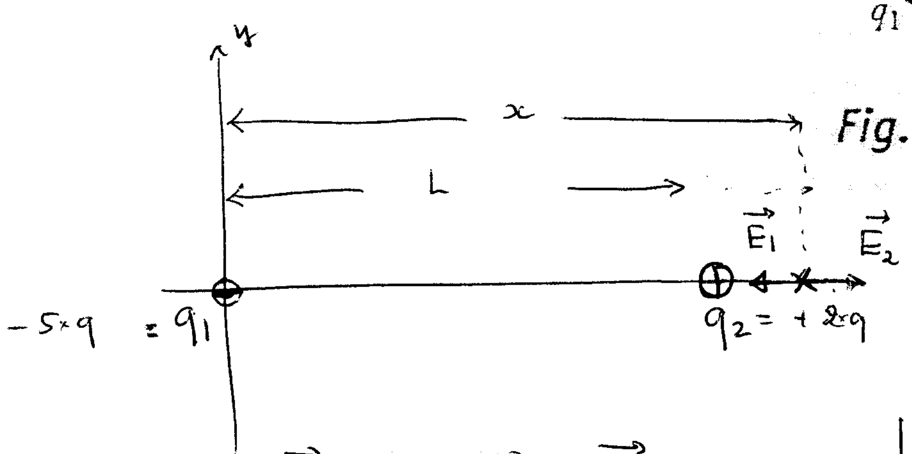


Fig. 22-30 Problem 8.



$$\vec{E}_{net} = \vec{E}_2 - \vec{E}_1 = 0 \Rightarrow |\vec{E}_2| = |\vec{E}_1|$$

$$\Rightarrow \frac{\cancel{k}|q_1|}{x^2} = \frac{\cancel{k}|q_2|}{(x-L)^2}$$

$$\frac{5\cancel{q}}{x^2} = \frac{2\cancel{q}}{(x-L)^2}$$

$$5(x-L)^2 = 2x^2$$

$$\frac{x-L}{x} = \sqrt{\frac{2}{5}}$$

$$1 - \frac{L}{x} = \sqrt{\frac{2}{5}}$$

••10) In Fig. 22-31, the four particles are fixed in place and have charges $q_1 = q_2 = +5e$, $q_3 = +3e$, and $q_4 = -12e$. Distance $d = 5.0\ \mu m$. What is the magnitude of the net electric field at point P due to the particles?

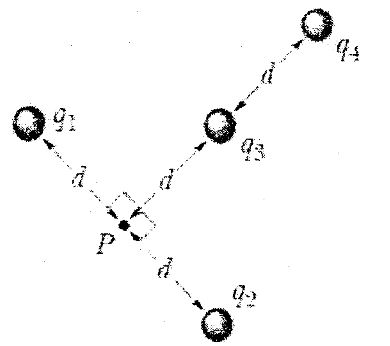
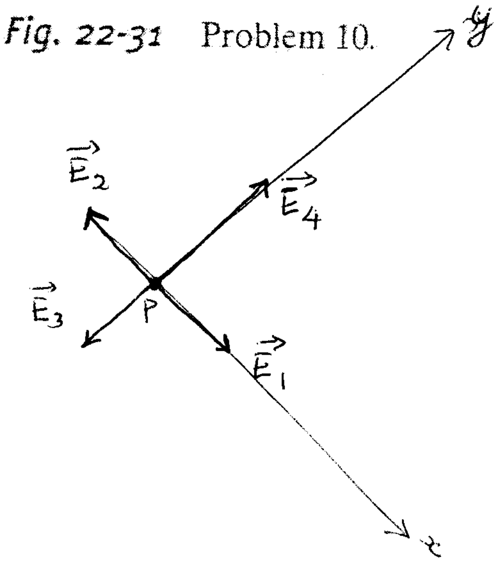


Fig. 22-31 Problem 10.

$$E_1 = \frac{k|q_1|}{r_1^2} = \frac{9\times10^{9}\times 5\times 1.6\times10^{-19}}{(5\times10^{-6})^2}$$

$$= 288\ N/C$$

$$E_2 = E_1 = 288\ N/C$$



$$E_3 = \frac{k|q_3|}{r_3^2} = \frac{9\times10^{9}\times 3\times1.6\times10^{-19}}{(5\times10^{-6})^2}$$

$$= 173\ N/C$$

$$E_4 = \frac{k|q_4|}{r_4^2} = \frac{9\times10^{9}\times 12\times1.6\times10^{-19}}{(10\times10^{-6})^2}$$

$$= 173\ N/C$$

$$\Rightarrow \vec{E}_{net} = (E_1 - E_2)\,\hat{i} + (E_4 - E_3)\,\hat{j} = (288-288)\,\hat{i} + (173-173)\,\hat{j}$$

$$= 0\,\hat{i} + 0\,\hat{j}$$

(••11) In Fig. 22-32, the four particles form a square of edge length $a = 5.00$ cm and have charges $q_1 = +10.0$ nC, $q_2 = -20.0$ nC, $q_3 = +20.0$ nC, and $q_4 = -10.0$ nC. In unit-vector notation, what net electric field do the particles produce at the square's center? SSM ILW WWW

$$E_1 = \frac{9\times10^{9} \times 10\times10^{-9}}{(0.035)^2} = 7.3\times10^{4}\ \frac{N}{C}$$

$$E_2 = \frac{9\times10^{9} \times 20\times10^{-9}}{(0.035)^2} = 14.7\times10^{4}\ \frac{N}{C}$$

$$E_3 = \frac{9\times10^{9} \times 20\times10^{-9}}{(0.035)^2} = 14.7\times10^{4}\ \frac{N}{C}$$

$$E_4 = 7.3\times10^{4}\ \frac{N}{C}$$

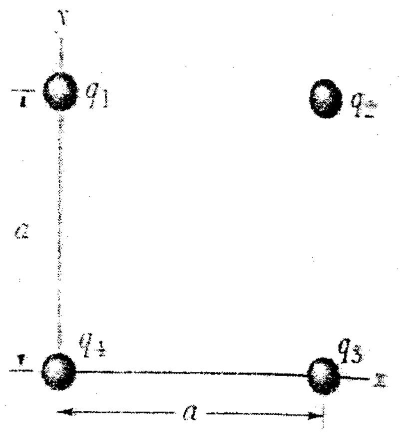
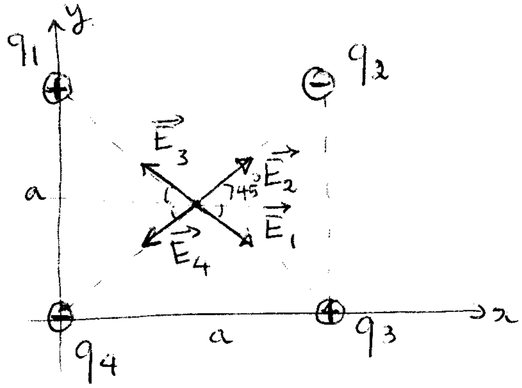


Fig. 22-32 Problem 11.

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$



$$E_{net,x} = E_1 \cos 45° + E_2 \cos 45° - E_3 \cos 45° - E_4 \cos 45°$$

$$= 0$$

$$r = \frac{a\sqrt{2}}{2} = 0.035\text{m}$$

$$E_{net,y} = E_2 \sin 45° + E_3 \sin 45° - E_1 \sin 45° - E_4 \sin 45°$$

$$= 2\times14.7\times10^{4} \sin 45° - 2\times7.3\times10^{4} \sin 45°$$

$$= 20.8\times10^{4} - 10.3\times10^{4} = 10.5\times10^{4}\ N/C$$

$$\boxed{\vec{E} = 0\hat{i} + 10.5\times10^{4}\hat{j}}\ N/C$$

•36 An electron is accelerated eastward at $1.80 \times 10^9\ \mathrm{m/s^2}$ by an electric field. Determine the (a) magnitude and (b) direction of the electric field.

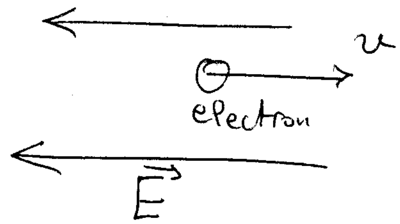
$$F = ma = q_0 E$$

$$\Rightarrow E = \frac{ma}{q_0}$$

$$= \frac{9.1 \times 10^{-3} \times 1.8 \times 10^{9}}{1.6 \times 10^{-19}}$$

magnitude $\boxed{E = 0.01\ \frac{N}{C}}$

direction



$\boxed{\vec{E} = -0.01\ \hat{i}\ \left(\frac{N}{C}\right)}$ Westward

••43 A uniform electric field exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2.0 cm away, in a time 1.5×10^{-8} s. (a) What is the speed of the electron as it strikes the second plate? (b) What is the magnitude of the electric field

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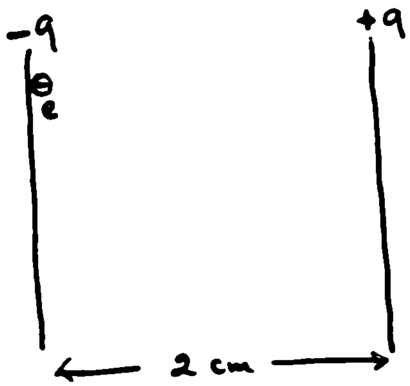


$x = 2$ cm

$t = 1.5 \times 10^{-8}$ s

$v_0 = 0$ (released from rest)

$m_e = 9.1 \times 10^{-31}$ Kg

a) From phys 101 $\Rightarrow$ $v = at + \cancel{v_0}$ — (1)

$x = \frac{1}{2} a t^2$ — (2)

(2) $\Rightarrow$ $a = \frac{2x}{t^2} = \frac{2 \times 0.02}{(1.5 \times 10^{-8})^2} = 1.8 \times 10^{14}\ m/s^2$

but $v = at = 1.8 \times 10^{14} \times 1.5 \times 10^{-8} = \boxed{2.7 \times 10^{6}\ m/s}$

b) From phys 102

$F = ma = |q| E$

$\Rightarrow E = \frac{ma}{|q|} = \frac{9.1 \times 10^{-31} \times 1.8 \times 10^{14}}{1.6 \times 10^{-19}} = 1 \times 10^{4}\ \frac{N}{C}$

this is the magnitude of the electric field.

From the figure $\boxed{\vec{E} = -1 \times 10^{4}\ \hat{i}\ \ N/C}$

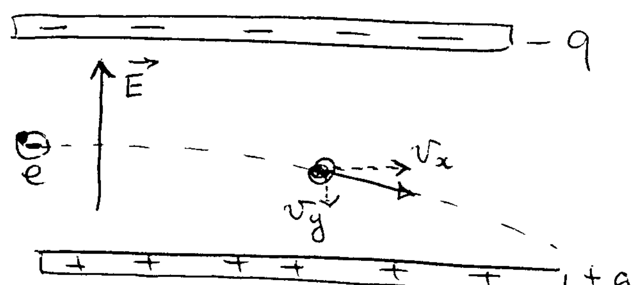
•44 At some instant the velocity components of an electron moving between two charged parallel plates are $v_x = 1.5 \times 10^5$ m/s and $v_y = 3.0 \times 10^3$ m/s. Suppose the electric field between the plates is given by $\vec{E} = (120\text{ N/C})\hat{\jmath}$. In unit-vector notation, what are (a) the electron's acceleration in that field and (b) the electron's velocity when its x coordinate has changed by 2.0 cm?

$v_x = 1.5 \times 10^5$ m/s

$= $ Constant

at some instant

$v_y = 3.0 \times 10^3$ m/s



a) $a = \dfrac{|q|E}{m} = \dfrac{1.6 \times 10^{-19} \times 120}{9.1 \times 10^{-31}} = 2.1 \times 10^{13}$ m/s²

$\vec{a} = 0\hat{\imath} - 2.1 \times 10^{13}\hat{\jmath}$ (m/s²)

b) $v_y = ?$ when $\Delta x = 2$ cm $= 0.02$m

x-axis $\Delta x = v_x t \Rightarrow t = \dfrac{\Delta x}{v_x}$

$t = \dfrac{0.02}{1.5 \times 10^5} = 1.3 \times 10^{-7}$ s

$v_y = v_{y_0} + a_y t = 3.0 \times 10^3 + 2.1 \times 10^{13} \times 1.3 \times 10^{-7}$

$v_y = 2.8 \times 10^6$ m/s

$$\boxed{\vec{v} = 1.5 \times 10^5\,\hat{\imath} - 2.8 \times 10^6\,\hat{\jmath} \;(\text{m/s})}$$