What phase difference between two otherwise identical traveling waves, moving in the same direction along a stretched string, will result in the combined wave having an amplitude 1.50 times that of the common amplitude of the two combining waves? Express your answer in (a) degrees, (b) radians, and (c) wavelengths.

$$y_1 = y_m \sin(kx - \omega t)$$

$$+ \quad y_2 = y_m \sin(kx - \omega t + \phi)$$

$$y' = \underbrace{2 y_m \cos\frac{\phi}{2}}_{y'_m} \sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

Given $y'_m = 1.5 \cancel{y_m} = 2 \cancel{y_m} \cos\frac{\phi}{2}$

$$\cos\frac{\phi}{2} = \frac{1.5}{2} = 0.75 \Rightarrow \frac{\phi}{2} = 0.72 \text{ rad}$$

b) $\Rightarrow \boxed{\phi = 1.45 \text{ rad}}$

a) $\phi = 1.45 \cancel{\text{rad}} \times \frac{180^\circ}{\pi \cancel{\text{rad}}} = \boxed{82.8^\circ}$

c) $\phi = \frac{2\pi}{\lambda} \Delta x$

$$\Delta x = \frac{\phi}{2\pi} \lambda = \boxed{0.23 \lambda}$$

•• A nylon guitar string has a linear density of 7.20 g/m and is under a tension of 150 N. The fixed supports are distance D = 90.0 cm apart. The string is oscillating in the standing wave pattern shown in Fig. 16-39. Calculate the (a) speed, (b) wavelength, and (c) frequency of the traveling waves whose superposition gives this standing wave. ILW

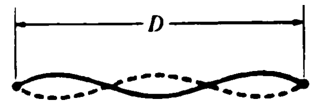


Fig. 16-39 Problem 43.

a) $v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{150}{7.2 \times 10^{-3}}} = \boxed{144.3 \text{ m/s}}$

b) $\frac{3\lambda}{2} = D \Rightarrow \lambda = \frac{2D}{3} = \frac{2 \times 0.9}{3} = \boxed{0.6 \text{ m}}$

c) $f = \frac{v}{\lambda} = \frac{144.3}{0.6} = \boxed{240.5 \text{ Hz}}$

One of the harmonic frequencies for a particular string under tension is 325 Hz. The next higher harmonic frequency is 390 Hz. What harmonic frequency is next higher after the harmonic frequency 195 Hz?

$$f_n = n\frac{v}{2L}$$ Harmonic frequencies. or resonant frequencies

$n = 1, 2, 3, \ldots$

$$f_n = n\frac{v}{2L} = 325 \text{ HZ}$$

$$f_{n+1} = (n+1)\frac{v}{2L} = 390 \text{ HZ}$$

$$f_1 = \frac{v}{2L}$$

$$f_{n+1} - f_n = \cancel{\frac{nv}{2L}} + \frac{v}{2L} - \cancel{\frac{nv}{2L}} = \frac{v}{2L} = f_1 =$$

$$390 - 325 = 65 \text{ HZ}$$

So $f_1 = 65$ HZ

$f_2 = 2f_1$, $f_3 = 3f_1 = f_2 + f_1$ etc...

$$f_{n+1} = f_n + f_1$$ $\Rightarrow$ next higher frequency is $195 + 65 = 260$ HZ

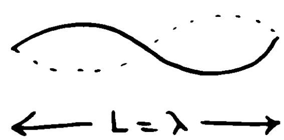
f_{n+1}: ? ; f_n: 195

$\boxed{260 \text{ HZ}}$

• A rope, under a tension of 200 N and fixed at both ends, oscillates in a second-harmonic standing wave pattern. The displacement of the rope is given by

$$y = (0.10\text{ m})(\sin \pi x/2) \sin 12\pi t,$$

where $x = 0$ at one end of the rope, x is in meters, and t is in seconds. What are (a) the length of the rope, (b) the speed of the waves on the rope, and (c) the mass of the rope? (d) If the rope oscillates in a third-harmonic standing wave pattern, what will be the period of oscillation?

a) 

2^{nd} harmonic $n = 2$

$$k = \frac{\pi}{2} = \frac{2\pi}{\lambda} \Rightarrow \lambda = 4\text{ m}$$

$$\Rightarrow \boxed{L = 4\text{ m}}$$

b) $$v = \frac{\omega}{k} = \frac{12\pi}{\frac{\pi}{2}} = \boxed{24\text{ m/s}}$$

c) $$v = \sqrt{\frac{\tau}{\mu}} \Rightarrow \mu = \frac{\tau}{v^2} = \frac{\text{mass}}{\text{length}} = \frac{m}{L}$$

$$\Rightarrow m = \frac{\tau L}{v^2} = \frac{200 \times 4}{(24)^2} = \boxed{1.4\text{ Kg}}$$

d) 3^{rd} harmonic

$$f = \frac{3v}{2L} = \frac{1}{T} \Rightarrow T = \frac{2L}{3v} = \boxed{0.11\text{ s}}$$

3^{rd} harmonic

$n = 3$

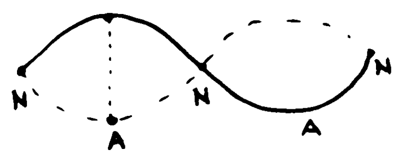
• The following two waves are sent in opposite directions on a horizontal string so as to create a standing wave in a vertical plane:

$$y_1(x, t) = (6.00\text{ mm})\sin(4.00\pi x - 400\pi t)$$

$$y_2(x, t) = (6.00\text{ mm})\sin(4.00\pi x + 400\pi t),$$

with x in meters and t in seconds. An antinode is located at point A. In the time interval that point takes to move from maximum upward displacement to maximum downward displacement, how far does each wave move along the string?

time $= \frac{T}{2}$



$$T = \frac{\omega}{2\pi} = \frac{400\pi}{2\pi} = 0.005\text{ s}$$

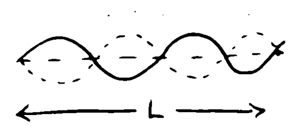
$$t = \frac{T}{2} = 0.0025\text{ s}$$

distance $\quad x = v\,t$

$$v = \frac{\omega}{k} = \frac{400\pi}{4\pi} = 100\text{ m/s}$$

$$\Rightarrow x = 100 \times 0.0025 = \boxed{0.25\text{ m}}$$

Oscillation of a 600 Hz tuning fork sets up standing waves in a string clamped at both ends. The wave speed for the string is 400 m/s. The standing wave has four loops and an amplitude of 2.0 mm. (a) What is the length of the string? (b) Write an equation for the displacement of the string as a function of position and time.



a) $L = 2\lambda = 2\dfrac{v}{f}$

$= 2\dfrac{400}{600} = \dfrac{8}{6} = \boxed{1.33\ m}$

b) $y' = 2y_m \sin kx \cos \omega t$

$y' = (4\ mm) \sin 9.4x \cos 1200\pi t$

$k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{0.67}$

$k = 9.4\ m^{-1}$

$\omega = 2\pi f$

$=$