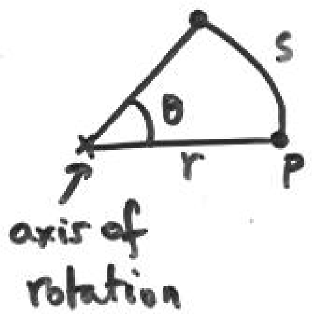
Chapter 11

* angular position $\theta \text{ (rad)} = \frac{s}{r}$

$$2\pi \text{ rad} = 360^\circ$$



$s = \theta r$

arc length

* angular displacement $\Delta\theta = \theta_2 - \theta_1$

* angular velocity $\bar{\omega} = \frac{\Delta\theta}{\Delta t} \left(\frac{\text{rad}}{\text{s}}\right)$

$$\omega_{inst} = \frac{d\theta}{dt} \left(\frac{\text{rad}}{\text{s}}\right)$$

* angular acceleration $\bar{\alpha} = \frac{\Delta\omega}{\Delta t} \left(\frac{\text{rad}}{\text{s}^2}\right)$

$$\alpha_{inst} = \frac{d\omega}{dt} \left(\frac{\text{rad}}{\text{s}^2}\right)$$

* Special Case when $\underline{\alpha = \text{Const.}}$

Kinematic Equations

$$\Delta\theta = \theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha\,\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)\,t$$

$$\Delta\theta = \omega t - \frac{1}{2}\alpha t^2$$

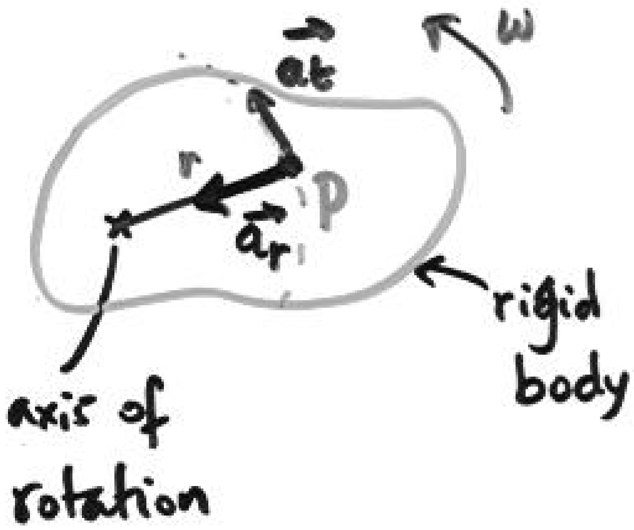
⇒ Relationship between linear and angular variables

$$s = \theta r$$

$$v = \frac{ds}{dt} = \omega r$$

$$a_t = \frac{dv}{dt} = \alpha r \quad \leftarrow \text{tangential acc.}$$

$$a_r = \frac{v^2}{r} = \omega^2 r \quad \leftarrow \text{radial acc.}$$



If $\omega = \text{Constant} \Rightarrow \alpha = \frac{d\omega}{dt} = 0$

$\Rightarrow a_t = \alpha r = 0$

There is only $a_r = \omega^2 r$

for example: a disk rotating at con
angular velocity of 5 rad/s.

If $\omega \neq \text{Const} \Rightarrow \alpha = \frac{d\omega}{dt} \neq 0$

$\Rightarrow a_t = \alpha r \neq 0$

There are two acc., a_r and a_t

for example: a disk rotates from $\omega =$

* Rotational Kinetic Energy

$$K = \frac{1}{2} I \omega^2$$

I: rotational inertia has unit ($kg \cdot m^2$)

* For discrete masses $I = \sum m_i r_i^2$

* For extended bodies $I = \int r^2 \, dm$

See Table 11.2 for I_{cm} of disk, sphere, hoop, thin rod, etc...

Parallel axis theorem

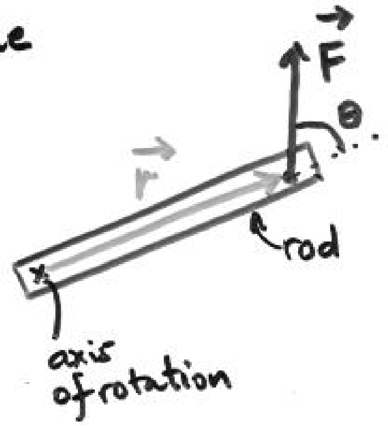
$$I = I_{cm} + M d^2$$

d: distance between axis given axis and through the center of mass.

example

$$I = I_{cm} + MR^2$$
$$= \frac{2}{5} MR^2 + MR^2 = \frac{7}{5} MR^2$$

* Torque



The force $\vec{F}$ will make the rod rotate.

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad (N.m)$$

- magnitude $\quad \tau = r F \sin\theta$

$\theta = 90^\circ \qquad \tau = r F$

$\theta = 0 \qquad \tau = 0$

- direction : use the right hand rule!

If many forces act on a rigid body to rotate it, then

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \ldots$$

Counter clockwise rotation $\Rightarrow \tau > 0$

Clockwise rotation $\Rightarrow \tau < 0$

* Newton's Second Law for Rotation

$$\tau_{net} = I \alpha$$

* Work and Rotational Kinetic Energy

⇒ Work energy theorem

$$\Delta K = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2 = W \quad \text{(Joule)}$$

If τ = constant

$$W = \tau (\theta_f - \theta_i) \quad \text{(Joule)}$$

Power

The rate at which the work is done is

$$P = \tau \omega \quad \text{(watt)}$$