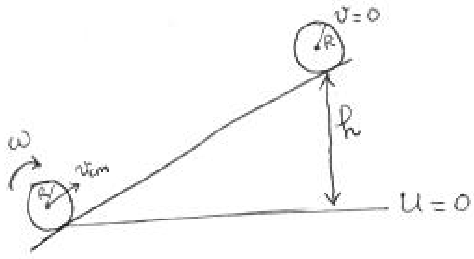
CHAPTER 12

6P. A body of radius R and mass m is rolling smoothly with speed v on a horizontal surface. It then rolls up a hill to a maximum height h. (a) If $h = 3v^2/4g$, what is the body's rotational inertia about the rotational axis through its center of mass? (b) What might the body be?



a) $\Delta K + \Delta U_g = 0$; $K_f = 0$ and $U_i = 0$

$$\Rightarrow \left[0 - \left(\frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} m v_{cm}^2\right)\right] + \left[mgh - 0\right] = 0$$

but $\omega = \dfrac{v_{cm}}{R} \Rightarrow \dfrac{1}{2} I \dfrac{v_{cm}^2}{R^2} + \dfrac{1}{2} m v_{cm}^2 = mg\dfrac{3v_{cm}^2}{4g}$

$$\frac{1}{2}\frac{I}{R^2} = -\frac{1}{2}m + \frac{3m}{4} = \frac{m}{4} \Rightarrow I = \frac{1}{2} m R^2$$

b) $\Rightarrow$ the object is a solid cylinder.

9P. A solid ball starts from rest at the upper end of the track shown in Fig. 12-31 and rolls without slipping until it rolls off the right-hand end. If $H = 6.0$ m and $h = 2.0$ m and the track is horizontal at the right-hand end, how far horizontally from point A does the ball land on the floor? ssm

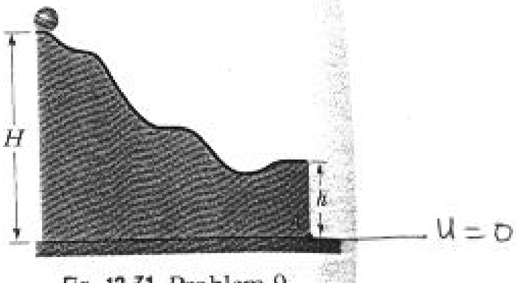


Fig. 12-31 Problem 9.

$$\Delta K + \Delta U_g = 0$$

$$K_i = 0$$

$$\left(\frac{1}{2} I \omega^2 + \frac{1}{2} m v^2 - 0\right) + \left(mg h - mg H\right) = 0$$

but $\omega = \frac{v_{cm}}{R}$ and $I = -\, m R^2$

$$\Rightarrow \frac{1}{2}\left(\frac{2}{5} m R^2 \frac{v_{cm}^2}{R^2}\right) + \frac{1}{2} m v_{cm}^2 = mg(H - h)$$

$$\frac{7}{10} \cancel{m} v_{cm}^2 = \cancel{m} g (H - h)$$

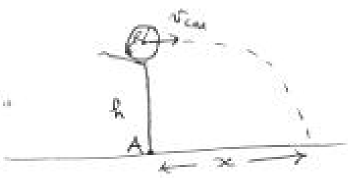
$$v_{cm} = \sqrt{\frac{10 g (H-h)}{7}}$$

$$v_{cm} = 7.5 \text{ m/s}$$

$$x = v_{cm} t$$

$$y = -h = -\frac{1}{2} g t^2$$

$$\Rightarrow t = \sqrt{\frac{2h}{g}} = 0.64 \text{ s}$$



$$\Rightarrow \boxed{x = 4.8 \text{ m}}$$

22P. What is the torque about the origin on a jar of jalapeño peppers located at coordinates (3.0 m, −2.0 m, 4.0 m) due to (a) force $\vec{F}_1 = (3.0\ \text{N})\hat{i} - (4.0\ \text{N})\hat{j} + (5.0\ \text{N})\hat{k}$, (b) force $\vec{F}_2 = (-3.0\ \text{N})\hat{i} - (4.0\ \text{N})\hat{j} - (5.0\ \text{N})\hat{k}$, and (c) the vector sum of $\vec{F}_1$ and $\vec{F}_2$? (d) Repeat part (c) about a point with coordinates (3.0 m, 2.0 m, 4.0 m) instead of about the origin.

3-Dim problem

a) $\vec{r} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ (m)

$\vec{F}_1 = 3\hat{i} - 4\hat{j} + 5\hat{k}$ (N)

$$\vec{\tau}_1 = \vec{r} \times \vec{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ 3 & -4 & 5 \end{vmatrix}$$

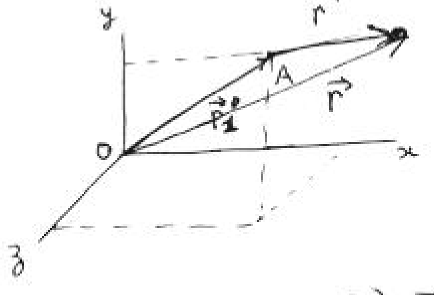
$$= (-10+16)\hat{i} - (15-12)\hat{j} + (-12+6)\hat{k}$$

$$\boxed{\vec{\tau}_1 = +6\hat{i} - 3\hat{j} - 6\hat{k}} \ (\text{N}\cdot\text{m})$$

b) $\vec{\tau}_2 = \vec{r} \times \vec{F}_2 = 26\hat{i} + 3\hat{j} - 18\hat{k}$ (N.m)

c) $\vec{\tau}_3 = \vec{r} \times (\vec{F}_1 + \vec{F}_2) = 32\hat{i} - 24\hat{k}$ (N.m)

d)



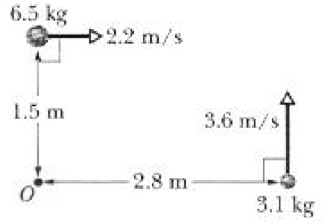
$\vec{r}_1 = 3\hat{i} + 2\hat{j} + 4\hat{k}$

$\vec{r} = 3\hat{i} - 2\hat{j} + 4\hat{k}$

$\vec{r}' = \vec{r} - \vec{r}_1 = -4\hat{j}$

$$\vec{\tau}_4 = \vec{r}' \times (\vec{F}_1 + \vec{F}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -4 & 0 \\ 0 & -8 & 0 \end{vmatrix} = 0$$

23E. Two objects are moving as shown in Fig. 12-35. What is their total angular momentum about point O? ilw



$$\vec{L} = \vec{l_1} + \vec{l_2}$$

$\vec{l_1}$ has a direction inside the page (use the right hand rule)

$\vec{l_2}$ has a direction outside the page

magnitudes :

$$l_1 = p_1 r_{1\perp} = m_1 v_1 r_{1\perp} = 6.5 \times 2.2 \times 1.5 = 21.45 \ Kg.\frac{m^2}{s}$$

$$l_2 = p_2 r_{2\perp} = m_2 v_2 r_{2\perp} = 3.1 \times 3.6 \times 2.8 = 31.25 \ Kg.\frac{m^2}{s}$$

$$L = l_2 - l_1 = 9.8 \ Kg \frac{m^2}{s}$$ It is out of the page!

24E. In Fig. 12-36, a particle P with mass 2.0 kg has position vector $\vec{r}$ of magnitude 3.0 m and velocity $\vec{v}$ of magnitude 4.0 m/s. A force $\vec{F}$ of magnitude 2.0 N acts on the particle. All three vectors lie in the xy plane oriented as shown. About the origin, what are (a) the angular momentum of the particle and (b) the torque acting on the particle?

2-Dim problem

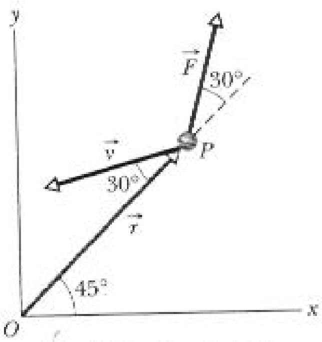
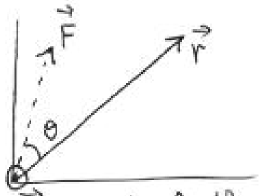


Fig. 12-36 Exercise 24.

b) $\tau = |\vec{r} \times \vec{F}| = r F \sin\theta$

$= 3 \times 2 \times \sin 30^\circ = 3 \quad N.m$



direction: $\vec{\tau}$ · out of the page

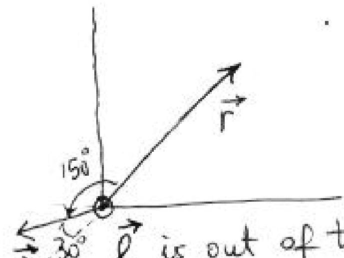
a)

$\vec{\ell} = \vec{r} \times \vec{P}$

$|\ell| = r\,p \sin\theta = m v r \sin\theta = 2 \times 4 \times 3 \times \sin 150^\circ$

$= 12 \text{ J.s}$

direction



$\vec{\ell}$ is out of the page

32P. At time $t = 0$, a 2.0 kg particle has position vector $\vec{r} = (4.0\text{ m})\hat{\text{i}} - (2.0\text{ m})\hat{\text{j}}$ relative to the origin. Its velocity just then is given by $\vec{v} = (-6.0t^2\text{ m/s})\hat{\text{i}}$. About the origin and for $t > 0$, what are (a) the particle's angular momentum and (b) the torque acting on the particle? (c) Repeat (a) and (b) about a point with coordinates $(-2.0\text{ m}, -3.0\text{ m}, 0)$ instead of about the origin.

a) $\vec{l} = \vec{r} \times \vec{p} = m\,(\vec{r} \times \vec{v})$

$$\vec{r} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 0 \\ -6t^2 & 0 & 0 \end{vmatrix} = 0\hat{i} - 0\hat{j} - 12t^2\,\hat{k}$$

$$\boxed{\vec{l} = (-24t^2)\,\hat{k}} \quad (\text{Kg} \cdot \text{m}^2/\text{s})$$

b) $\vec{\tau} = \vec{r} \times \vec{F} = m\,(\vec{r} \times \vec{a}) \quad [\vec{F} = m\vec{a}]$

$$\vec{a} = \frac{d\vec{v}}{dt} = -12t\,\hat{i} \quad (\text{m/s}^2)$$

$$\vec{r} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 0 \\ -12t & 0 & 0 \end{vmatrix} = -24t\,\hat{k}$$

$$\boxed{\vec{\tau} = -48t\,\hat{k} \quad (\text{N} \cdot \text{m})}$$

38P. Figure 12-39 shows a rigid structure consisting of a circular hoop of radius R and mass m, and a square made of four thin bars, each of length R and mass m. The rigid structure rotates at a constant speed about a vertical axis, with a period of rotation of 2.5 s. Assuming $R = 0.50$ m and $m = 2.0$ kg, calculate (a) the structure's rotational inertia about the axis of rotation and (b) its angular momentum about that axis.

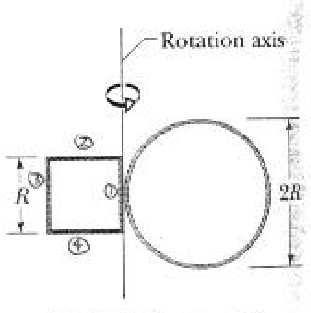


Fig. 12-39 Problem 38.

a) $I = I_{hoop} + I_{rods}$

$I_{hoop} = \frac{1}{2} m R^2 + m R^2 = \frac{3}{2} m R^2$

$I_{rods} = \underset{\text{rod 1}}{0} + \underbrace{(mR^2 + 0)}_{\text{rod 3}} + \underbrace{\frac{1}{3} m R^2}_{\text{rod 2}} + \underbrace{\frac{1}{3} m R^2}_{\text{rod 4}} = \frac{5}{3} m R^2$

$I_{Total} = \frac{19}{6} m R^2 = 1.6 \text{ Kg} \cdot m^2$

b) $L = I\omega = \frac{19}{6} m R^2 \omega$

period = 2.5 s (time for One complete rotation)

$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{2.5 \text{ sec}} = 2.5 \text{ rad/s}$

$\boxed{L = 4.0 \text{ Kg } \frac{m^2}{s}}$

39E. A man stands on a platform that is rotating (without friction) with an angular speed of 1.2 rev/s; his arms are outstretched and he holds a brick in each hand. The rotational inertia of the system consisting of the man, bricks, and platform about the central axis is 6.0 kg · m². If by moving the bricks the man decreases the rotational inertia of the system to 2.0 kg · m², (a) what is the resulting angular speed of the platform and (b) what is the ratio of the new kinetic energy of the system to the original kinetic energy? (c) What provided the added kinetic energy? ssm

a) Angular momentum is conserved because there are no external torques $\Rightarrow L_i = L_f$

or $I_i \omega_i = I_f \omega_f$

$I_i = 6 \text{ Kg m}^2$ $\omega_i = 1.2 \text{ rev/s} = 7.5 \text{ rad/s}$

$I_f = 2 \text{ Kg m}^2$

$$\Rightarrow \omega_f = \frac{I_i}{I_f}\omega_i = \boxed{22.6 \text{ rad/s}}$$

b) $K_f = \frac{1}{2} I_f \omega_f^2$

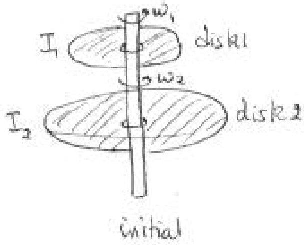
$K_i = \frac{1}{2} I_i \omega_i^2$

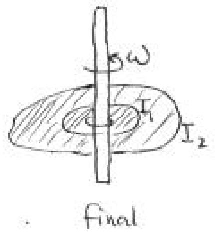
$$\Longrightarrow \frac{K_f}{K_i} = \frac{I_f \omega_f^2}{I_i \omega_i^2} = \frac{2 \times (22.6)^2}{6 \times (7.5)^2} = \boxed{3}$$

c) Since $K_f = 3 K_i$ there is increase in Kinetic energy. This increase is provided by the man himself.

42E. Two disks are mounted on low-friction bearings on the same axle and can be brought together so that they couple and rotate as one unit. (a) The first disk, with rotational inertia $3.3 \text{ kg} \cdot \text{m}^2$ about its central axis, is set spinning at 450 rev/min. The second disk, with rotational inertia $6.6 \text{ kg} \cdot \text{m}^2$ about its central axis, is set spinning at 900 rev/min in the same direction as the first. They then couple together. What is their angular speed after coupling? (b) If instead the second disk is set spinning at 900 rev/min in the direction opposite the first disk's rotation, what is their angular speed and direction of rotation after coupling?

a)





$I_1 = 3.3 \text{ Kg}\cdot\text{m}^2$ $\qquad \omega_1 = 450 \frac{\text{rev}}{\text{min}} = 47.1 \text{ rad/s}$

$I_2 = 6.6 \text{ Kg}\cdot\text{m}^2$ $\qquad \omega_2 = 900 \frac{\text{rev}}{\text{min}} = 94.2 \text{ rad/s}$

$$L_i = L_f \Rightarrow I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$$

$$\Rightarrow \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \boxed{78.5 \text{ rad/s}}$$

b)

$$L_i = L_f \Rightarrow I_1\omega_1 - I_2\omega_2 = (I_1 + I_2)\omega$$

$$\omega = \frac{I_1\omega_1 - I_2\omega_2}{I_1 + I_2} = \boxed{-47.1 \text{ rad/s}}$$

It is the same direction as disk 2.

50P. A uniform thin rod of length 0.50 m and mass 4.0 kg can rotate in a horizontal plane about a vertical axis through its center. The rod is at rest when a 3.0 g bullet traveling in the horizontal plane of the rod is fired into one end of the rod. As viewed from above, the direction of the bullet's velocity makes an angle of 60° with the rod (Fig. 12-42). If the bullet lodges in the rod and the angular velocity of the rod is 10 rad/s immediately after the collision, what is the bullet's speed just before impact?

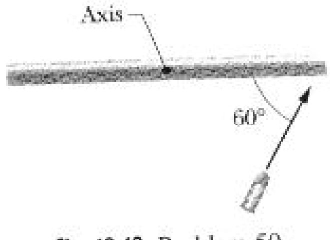
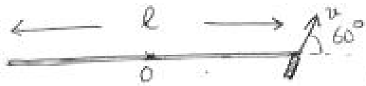
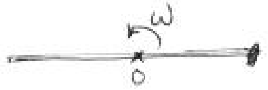


Fig. 12-42 Problem 50.



before



after

Conservation of angular momentum $L_i = L_f$

$$mv\left(\frac{\ell}{2}\right)\sin 60^\circ + 0 = \left[\frac{1}{12}M\ell^2 + m\left(\frac{\ell}{2}\right)^2\right]\omega$$

velocity of the bullet before Collision

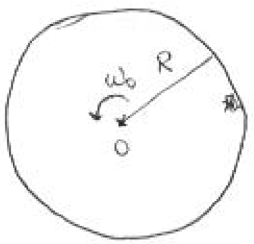
angular velocity of (rod + bullet) after Collision

$$v = \frac{\frac{1}{12}M\ell^2 + m\left(\frac{\ell}{2}\right)^2}{m\left(\frac{\ell}{2}\right)\sin 60^\circ} = \frac{\left[\frac{1}{12}(4)(0.5)^2 + (0.003)(0.25)^2\right]\times 10}{(0.003)(0.25)\sin 60^\circ}$$

$$\boxed{v = 1286 \text{ m/s}}$$

52P. A cockroach of mass m lies on the rim of a uniform disk of mass $10.0m$ that can rotate freely about its center like a merry-go-round. Initially the cockroach and disk rotate together with an angular velocity of ω_0. Then the cockroach walks halfway to the center of the disk. (a) What is the change $\Delta\omega$ in the angular velocity of the cockroach–disk system? (b) What is the ratio K/K_0 of the new kinetic energy of the system to its initial kinetic energy? (c) What accounts for the change in the kinetic energy?

a)



overhead view

a) Conservation of angular momentum $L_i = L_f$

$$\left(I_{disk} + m R^2\right) \underset{\omega_0}{\underset{\|}{\omega_i}} = \left[I_{disk} + m \left(\frac{R}{2}\right)^2\right] \omega_f$$

$$\Delta\omega = \omega_f - \omega_0 = \left[\frac{\frac{1}{2}(10m)R^2 + m R^2}{\frac{1}{2}(10m)R^2 + m\frac{R^2}{4}} - 1\right] \omega_0$$

$$= \left(\frac{5m + m}{5m + \frac{m}{4}} - 1\right) \omega_0 = (1.14 - 1)\omega_0$$

$$\boxed{\Delta\omega = 0.14\,\omega_0}$$

b)

$$\frac{K_f}{K_0} = \frac{\frac{1}{2} I_f \omega_f^2}{\frac{1}{2} I_i \omega_i^2} = \frac{\frac{1}{2} \overbrace{(I_f \omega_f)}^{L_f} \omega_f}{\frac{1}{2} \underbrace{(I_i \omega_i)}_{L_i} \omega_i} = \frac{\omega_f}{\omega_i} = \boxed{1.14}$$

$$\boxed{K_f = 1.14 \underset{K_i}{\underset{\|}{K_0}}}$$

c) The increase in K.E. is due to cockroach who does positive work in moving from R to $\frac{R}{2}$.

57P. In Fig. 12-45, a 1.0 g bullet is fired into a 0.50 kg block that is mounted on the end of a 0.60 m nonuniform rod of mass 0.50 kg. The block–rod–bullet system then rotates about a fixed axis at point *A*. The rotational inertia of the rod alone about *A* is 0.060 kg · m². Assume the block is small enough to treat as a particle on the end of the rod. (a) What is the rotational inertia of the block–rod–bullet system about point *A*? (b) If the angular speed of the system about *A* just after the bullet's impact is 4.5 rad/s, what is the speed of the bullet just before the impact?

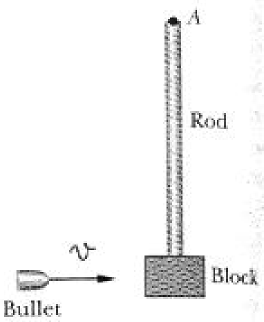


Fig. 12-45 Problem 57.

a) $I = I_{rod} + I_{block} + I_{bullet} = 0.06 + 0.5\times(0.6)^2 + (0.001)\times(0.6)^2$

$I = 0.24\ Kg.m^2$

b) $\omega_f = 4.5\ rad/s$

$v = ?$

Conservation of L $\Rightarrow$ $L_i = L_f$

$m_b v_b r = I \omega_f$

$v_b = \dfrac{I\omega_f}{m_b r} = \dfrac{(0.24)(4.5)}{0.001 \times 0.6} = \boxed{1803\ m/s}$