Summary of chapter 14

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➢ The gravitational force:

The magnitude of the gravitational force between two **particles** of mass m_1 and m_2 is given by

$$F_{12} = G\frac{m_1 m_2}{r^2} = F_{21}$$

Where G is the gravitational constant = 6.65 x 10^{-11} N.m^2/kg^2 is a constant for the whole universe and r is the distance between the two particles.

If the particles are replaced by rigid bodies, then r is the distance from center to center of the two bodies.

➢ Principle of superposition:

Here we have more than two objects. The force on one of the object is the vector sum of the forces due to the remaining objects.

$$F_1 = \sum_{i=2}^{n} F_{1i} = F_{12} + F_{13} + F_{14} + \ldots + F_{1n}$$

Be careful: This is a vector equation. F_1 is the force on particle 1 due to the other particles and $F_{1i} = G\frac{m_1 m_i}{r_{1i}^2}$

➢ Gravitation near the surface of the Earth:

Suppose a particle of mass m is at an altitude h above the surface of the Earth, then there is a force on the particle given by $F = G\frac{M_E m}{r^2} = ma_g$ where a_g is the gravitational acceleration. Then a_g is given by:

$$a_g = G\frac{M_E}{r^2}$$

where M_E is the mass of the Earth and $r = R_E + h$. Here h is the distance from the surface of the Earth to the particle.

Because the **Earth rotates**, then the difference between the free fall acceleration g and a_g is given by

$$a_g - g = \left(\frac{2\pi}{T}\right)^2 R$$

where T is the period of rotation of the Earth and R is the perpendicular distance between the object (on the surface of the Earth) and the axis of rotation of the Earth about itself.

- ✓ **At the equator** R is R_E and a_g $g = 0.034\ m/s^2$.
- ✓ **At the poles** R is zero and a_g $g = 0$, that is $a_g = g$.

Gravitation inside the Earth:

The gravitational force on a particle of mass m inside the Earth is given by (as was proved in the lecture):

$$F = G\frac{M' m}{r^2} = G\left(\frac{4}{3}\pi r^3 \rho\right)\frac{m}{r^2} = \left(\frac{4\pi m G}{3}\rho\right)r$$

where r is the distance from the center of the Earth to the particle and ρ is the density of the Earth.

The gravitational potential energy:

The potential energy between two particles of mass m_1 and m_2 separated by a distance r is given by:

$$U = -\frac{G m_1 m_2}{r}$$

the unit is **Joule** and we take U at infinity to be zero.

If the system consists of **three particles** m_1, m_2 and m_3, then U will be:

$$U = -G\left(\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_2 m_3}{r_{23}}\right)$$

- For **four particles system** you will have **6 terms**.

- Suppose you want fire a projectile of mass m from the surface of the Earth in a distance r in space. The total energy is conserved in this process.

$$E_i = K_i + U_i = E_f = K_f + U_f$$

or
$$\frac{1}{2}mv_i^2 - G\frac{Mm}{R_E} = \frac{1}{2}mv_f^2 - G\frac{Mm}{r}$$

If the projectile is to move upward forever, then the initial speed is called the **escape speed**.

In this case set $v_I = v_{esc}$, $r = \infty$, $v_f = 0 \Rightarrow K_f = 0$ and $U_f = 0$. Then the escape speed will be given by:

$$v_{esc} = \sqrt{2\frac{GM}{R_E}}$$

Note: ***The escape speed does not depend of the mass of the projectile. It depends only on the mass and radius of the planet from which you fire the projectile.***

Kepler s laws:

- The law of orbits: All planets move in **elliptical orbits** with the Sun at one focus.

- ❖ The angular momentum of the planet, with the Sun as the origin, **is conserved**.

- ❖ The period of rotation of any planet around the Sun is related to **the semi-major axis a** by the relation:

$$T^2 = \left(\frac{4\pi^2}{GM}\right)a^3$$

where M is the mass of the Sun in this case.

For circular orbits, replace a by r, the radius of the orbit.

➢ Energies in Satellite motion:

Suppose a satellite orbits the **Earth** at a distance r from the center of the Earth in a circular orbit. The satellite will have an **orbital speed** of

$$v = \sqrt{\frac{GM}{r}}$$

The kinetic energy of a satellite is :

$$K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

and the potential energy is:

$$U = -\frac{GMm}{r}$$

The total mechanical energy of the satellite at this location will be:

$$E = K + U = -\frac{GMm}{2r}$$

Here M is the mass of the Earth and m is the mass of the satellite. r is the distance from the satellite to the center of the Earth.

For ***elliptical orbits***, the mechanical energy is:

$$E = -\frac{GMm}{2a}$$

where a is the semi-major axis.