<u>Summary of chapter 12</u>

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- **Rolling = Pure Rotation** about the center of mass + **Pure translation** of the center of mass

During the rolling motion the linear velocity of the center of mass is given by:

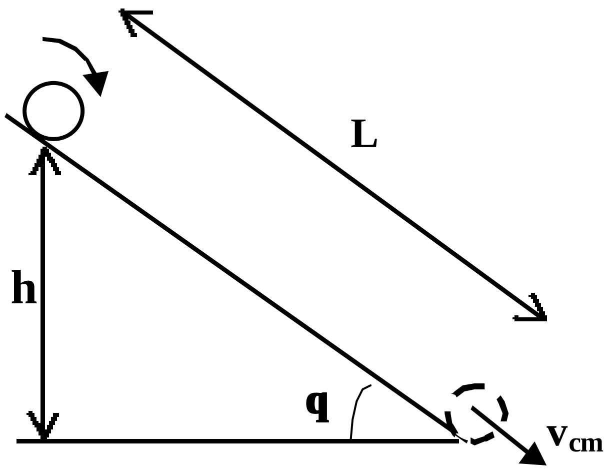
$$v_{cm} = \omega R$$

where R is the radius of the rolling object and ω is its angular velocity.

The velocity of the top of the rolling object is given by:

$$v_{top} = 2\omega R$$

- **An example of rolling motion:**



<u>Note that:</u>

1. The force of friction is responsible for the rolling motion.
2. The friction force does no work in this case.
3. The weight and the normal force do not produce torques.

- You can use either **<u>conservation of mechanical energy</u>** or ______________________ to calculate the velocity of the center of mass at the bottom of the incline. It is given by:

$$v_{cm} = \sqrt{\frac{2gL\sin\theta}{1+\frac{I_{cm}}{MR^2}}}$$

Consider a hoop, a disk and a cylinder of same mass and same radius. Then the larger v_{cm} will that of the the object with the smallest I_{cm}.

Since $(I_{cm})_{sphere} < (I_{cm})_{disk} < (I_{cm})_{hoop}$

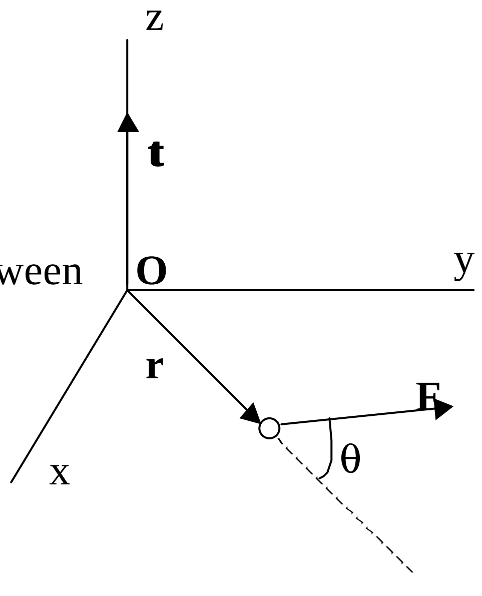
Then $(v_{cm})_{sphere} > (v_{cm})_{disk} > (v_{cm})_{hoop}$

- The torque, **about the origin**, acting on a particle is given by:

$$\vec{\tau} = \vec{r} \times \vec{F}$$ The unit is (N.m)

- The **magnitude** of the torque is:

$$\tau = rF\sin\theta$$



where r is **the position vector** of the particle **relative to the origin O** and θ is the angle between r and F.

- The direction is found using **the right hand rule**.

  In this figure, the torque is along the z axis if r and F are both in the x-y plane.

If you are given the vectors r and F in **unit vector notation**, then the torque can be found as:

$$\vec{\tau} = (yF_z - zF_y)\hat{i} - (xF_z - zF_x)\hat{j} + (xF_y - yF_x)\hat{k}$$

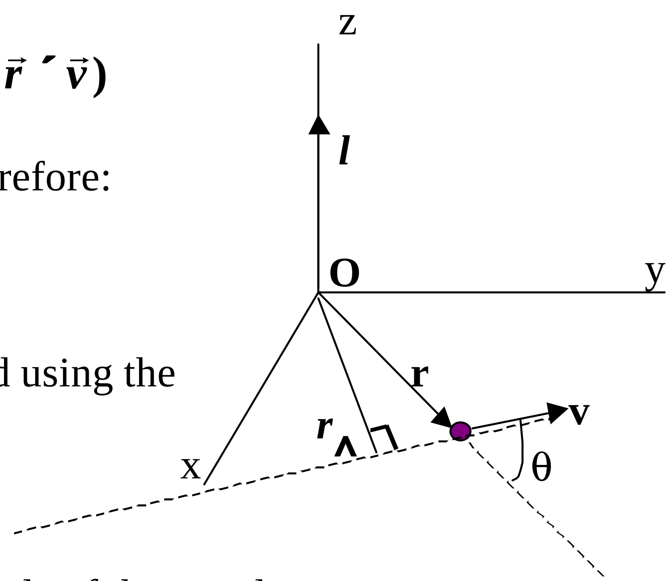
- The **angular momentum** for a single particle of mass m and moving with a velocity v is defined as:

$$\vec{l} = (\vec{r} \times \vec{p}) = m(\vec{r} \times \vec{v})$$

The magnitude of *l* is therefore:

$$l = mrv\sin\theta$$

and the direction is found using the right hand rule.



We can write the magnitude of the angular momentum as:

$$l = mvr_{\perp}$$

where $r_{\perp}$ = r sinθ. This is the perpendicular distance between the origin and the extension of the vector v (see the figure).

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for the linear motion and is given by:

For a **single particle**: $$\sum \vec{\tau} = \frac{d\vec{l}}{dt}$$

For a **system of particles**: $$\sum \vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

and

$$\vec{L} = \vec{l}_1 + \vec{l}_2 + \vec{l}_3 + \vec{l}_4 + \cdots$$

- In the case of a **rigid body, rotating about a fixed axis** (call it the z axis), then:

$$L_z = Iw$$

where I is the rotational inertia about that axis and w is the angular velocity.

- Conservation of angular momentum:

We know that $$\sum \vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

- If the <u>net torque</u> acting on a particle <u>is zero</u>, then $\frac{d\vec{L}}{dt} = 0$ and the **angular momentum** **<u>remains constant</u>** or **<u>is conserved</u>**.

In this case; we have: $$L_i = L_f$$

- If the <u>net torque</u> acting on a particle is <u>NOT zero</u>, then the **angular momentum** **<u>changes with time</u>** and is therefore **<u>NOT conserved</u>**.