FORMULA SHEET – PHYS 301

Useful formulas

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin 2A = 2 \sin A \cos A$

$e^{i\theta} = \cos\theta + i\sin\theta$

$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right), \quad \int \frac{xdx}{a^2 - x^2} = -\frac{1}{2}\ln(a^2 - x^2)$

$\int \frac{dx}{a^2x^2 - b^2} = \frac{1}{2ab}\ln(\frac{ax-b}{ax+b})$

$\int e^{ax}\sin(x)dx = \frac{e^{ax}}{a^2+1}(a\sin x - \cos x)$

$\int e^{ax}\sin^2(x)dx = \frac{e^{ax}}{a^2+4}(a\sin^2 x - 2\sin x\cos x + \frac{2}{a})$

Chapter 4

$F(x) \cong -kx + \varepsilon x^3 \quad U(x) = \frac{1}{2}kx^2 - \frac{1}{4}\varepsilon x^4$

$\dot{x} \propto \sqrt{E - U(x)}$

$\ddot{x} + \mu(x^2 - a^2)\dot{x} + \omega_0^2 x = 0$

Chapter 5

$\mathbf{F} = -G\frac{mM}{r^2}\mathbf{e_r}, \quad G = 6.67 \times 10^{-11} N.m^2/kg^2$

$\mathbf{F} = -Gm\int_V \frac{\rho(\mathbf{r}')\mathbf{e_r}}{r^2}dV', \quad \mathbf{g} = -G\int_V \frac{\rho(\mathbf{r}')\mathbf{e_r}}{r^2}dV'$

$\mathbf{g} = -\nabla\Phi, \quad \Phi = -G\int_V \frac{\rho(\mathbf{r}')}{r}dV'$

Chapter 7

$\delta\int_{t_1}^{t_2} L(q_j, \dot{q}_j, t)dt = 0$

$\frac{\partial L}{\partial q_j} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} = 0, \qquad j = 1, 2, \ldots, s$

$\frac{\partial L}{\partial q_j} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} + \sum_k \lambda_k(t)\frac{\partial f}{\partial q_j} = 0, \qquad f(q_j, \dot{q}_j, t) = 0$

$p_j = \frac{\partial L}{\partial \dot{q}_j}, \quad \dot{p}_j = \frac{\partial L}{\partial q_j}$

$H(q_k, p_k, t) = \sum_j p_j\dot{q}_j - L(q_k, \dot{q}_k, t)$

$\dot{q}_k = \frac{\partial H}{\partial p_k}, \qquad -\dot{p}_k = \frac{\partial H}{\partial q_k}$

Chapter 2

$\mathbf{p} = m\mathbf{v}, \quad \mathbf{F} = m\ddot{\mathbf{r}}$

$\mathbf{L} = \mathbf{r} \times \mathbf{p}, \quad \mathbf{N} = \mathbf{r} \times \mathbf{F}$

$W_{12} = \int_1^2 \mathbf{F} \cdot d\mathbf{r} = U_1 - U_2$

$\mathbf{F} = -\nabla U, \quad E = T + U$

$\left(\frac{dx}{dt}\right)^2 = \pm\sqrt{\frac{2}{m}[E - U(x)]}$

Chapter 3

$F = -kx$

$\ddot{x} + \omega_0^2 x = 0, \quad \omega_0^2 = \frac{k}{m}$

$\tau_0 = 2\pi\sqrt{\frac{k}{m}} = \frac{1}{\nu_0}$

$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0 \rightarrow x(t) = e^{-\beta t}[A_1 e^{\sqrt{\beta^2-\omega_0^2}t} + A_2 e^{-\sqrt{\beta^2-\omega_0^2}t}]$

$x(t) = Ae^{-\beta t}\cos(\omega_1 t - \delta) \quad x(t) = e^{-\beta t}(A + Bt)$

$x(t) = e^{-\beta t}[A_1 e^{\omega_2 t} + A_2 e^{-\omega_2 t}]$

$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos\omega t \rightarrow x(t) = x_c(t) + x_p(t)$

$x_p(t) = D\cos(\omega t - \delta)$

$D = \frac{A}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2}}$

$\delta = \tan^{-1}(\frac{2\omega\beta}{\omega_0^2 - \omega^2}), \quad \text{Q} = \frac{\omega_R}{2\beta} \quad \text{Q} = \frac{\omega_o}{\Delta\omega}$

$x_{en} = \pm Ae^{-\beta t} \quad decrement = e^{\beta\tau_1}$

$V = Ri \quad V = L\frac{di}{dt} \quad V = \frac{q}{C}$

$\omega_{\max} = \frac{1}{\sqrt{LC - \frac{R^2C^2}{2}}}$

$P = I^2R; \quad P = Q^2/2C; \quad P = LI^2/2$

Chapter 6

$J = \int_{x_1}^{x_2} f\{y_i(x), y_i'(x); x\}dx, \quad i = 1, 2, \ldots, n$

$\frac{\partial f}{\partial y_i} - \frac{d}{dx}\frac{\partial f}{\partial y_i'} = 0, \quad i = 1, 2, \ldots, n$

$f - y'\frac{\partial f}{\partial y'} = \text{constant} \quad (for \ \frac{\partial f}{\partial x} = 0)$

$\frac{\partial f}{\partial y_i} - \frac{d}{dx}\frac{\partial f}{\partial y_i'} + \sum_j \lambda(x)\frac{\partial g_j}{\partial y_i} = 0, \quad g_j\{y_i(x); x\} = 0$

Chapter 8

$L = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2) - U(r)$

$\ell = \mu r^2\dot{\theta} = \text{constant}$

$E = \frac{1}{2}\mu\dot{r}^2 + \frac{1}{2}\frac{\ell^2}{\mu r^2} + U(r)$

$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{\ell^2}F(r)$

$V(r) = U(r) + \frac{\ell^2}{2\mu r^2}$