1
Relativity I

Chapter Outline

At the end of the 19th century, scientists believed that they had learned most of what there was to know about physics. Newton's laws of motion and his universal theory of gravitation, Maxwell's theoretical work in unifying electricity and magnetism, and the laws of thermodynamics and kinetic theory employed mathematical methods to successfully explain a wide variety of phenomena.

However, at the turn of the 20th century, a major revolution shook the world of physics. In 1900 Planck provided the basic ideas that led to the quantum theory, and in 1905 Einstein formulated his special theory of relativity. The excitement of the times is captured in Einstein's own words: "It was a marvelous time to be alive." Both ideas were to have a profound effect on our understanding of nature. Within a few decades, these theories inspired new developments and theories in the fields of atomic, nuclear, and condensed-matter physics.

Although modern physics has led to a multitude of important technological achievements, the story is still incomplete. Discoveries will continue to be made during our lifetime, many of which will deepen or refine our understanding of nature and the world around us. It is still a "marvelous time to be alive."

1.1 SPECIAL RELATIVITY

Light waves and other forms of electromagnetic radiation travel through free space at the speed $c = 3.00 \times 10^8$ m/s. As we shall see in this chapter, the speed of light sets an upper limit for the speeds of particles, waves, and the transmission of information.

Most of our everyday experiences deal with objects that move at speeds much less than that of light. Newtonian mechanics and early ideas on space and time were formulated to describe the motion of such objects, and this formalism is very successful in describing a wide range of phenomena. Although Newtonian mechanics works very well at low speeds, it fails when applied to particles whose speeds approach that of light. Experimentally, one can test the predictions of Newtonian theory at high speeds by accelerating an electron through a large electric potential difference. For example, it is possible to accelerate an electron to a speed of $0.99c$ by using a potential difference of several million volts. According to Newtonian mechanics, if the potential difference (as well as the corresponding energy) is increased by a factor of 4, then the speed of the electron should be doubled to $1.98c$. However, experiments show that the speed of the electron—as well as the speeds of all other particles in the universe—always remains less than the speed of light, regardless of the size of the accelerating voltage. In part because it places no upper limit on the speed that a particle can attain, Newtonian mechanics is contrary to modern experimental results and is therefore clearly a limited theory.

In 1905, at the age of 26, Albert Einstein published his *special theory of relativity*. Regarding the theory, Einstein wrote,

> The relativity theory arose from necessity, from serious and deep contradictions in the old theory from which there seemed no escape. The strength of the new theory lies in the consistency and simplicity with which it solves all these difficulties, using only a few very convincing assumptions. . . .[1]

Although Einstein made many important contributions to science, the theory of relativity alone represents one of the greatest intellectual achievements of the 20th century. With this theory, one can correctly predict experimental observations over the range of speeds from rest to speeds approaching the speed of light. Newtonian mechanics, which was accepted for over 200 years, is in fact a limiting case of Einstein's special theory of relativity. This chapter and the next give an introduction to the special theory of relativity, which deals with the analysis of physical events from coordinate systems moving with constant speed in straight lines with respect to one another. Chapter 2 also includes a short introduction to general relativity, which describes physical events from coordinate systems undergoing general or accelerated motion with respect to each other.

In this chapter we show that the special theory of relativity follows from two basic postulates:

1. The laws of physics are the same in all reference systems that move uniformly with respect to one another. That is, basic laws such as

[1] A. Einstein and L. Infeld, *The Evolution of Physics,* New York, Simon and Schuster, 1961.

$\Sigma \mathbf{F} = d\mathbf{p}/dt$ have the same mathematical form for all observers moving at constant velocity with respect to one another.

2. The speed of light in vacuum is always measured to be 3×10^8 m/s, and the measured value is independent of the motion of the observer or of the motion of the source of light. That is, the speed of light is the same for all observers moving at constant velocities.

Although it is well known that relativity plays an essential role in theoretical physics, it also has practical applications, for example, in the design of particle accelerators, global positioning system (GPS) units, and high-voltage TV displays. Note that these devices simply will not work if designed according to Newtonian mechanics! We shall have occasion to use the outcomes of relativity in many subsequent topics in this text.

1.2 THE PRINCIPLE OF RELATIVITY

To describe a physical event, it is necessary to establish a frame of reference, such as one that is fixed in the laboratory. Recall from your studies in mechanics that Newton's laws are valid in inertial frames of reference. *An inertial frame is one in which an object subjected to no forces moves in a straight line at constant speed*—thus the name "inertial frame" because an object observed from such a frame obeys Newton's first law, the law of inertia.[2] Furthermore, any frame or system moving with constant velocity with respect to an inertial system must also be an inertial system. Thus there is no single, preferred inertial frame for applying Newton's laws.

Inertial frame of reference

According to the **principle of Newtonian relativity,** the laws of mechanics must be the same in all inertial frames of reference. For example, if you perform an experiment while at rest in a laboratory, and an observer in a passing truck moving with constant velocity performs the same experiment, Newton's laws may be applied to both sets of observations. Specifically, in the laboratory or in the truck a ball thrown up rises and returns to the thrower's hand. Moreover, both events are measured to take the same time in the truck or in the laboratory, and Newton's second law may be used in both frames to compute this time. Although these experiments look different to different observers (see Fig. 1.1, in which the Earth observer sees a different path for the ball) and the observers measure different values of position and velocity for the ball at the same times, both observers agree on the validity of Newton's laws and principles such as conservation of energy and conservation of momentum. This implies that no experiment involving mechanics can detect any essential difference between the two inertial frames. The only thing that can be detected is the relative motion of one frame with respect to the other. That is, the notion of *absolute* motion through space is meaningless, as is the notion of a single, preferred reference frame. **Indeed, one of the firm philosophical principles of modern science is that all observers are equivalent and that the laws of nature must take the same mathematical form for all observers.** Laws of physics that exhibit the same mathematical form for observers with different motions at different locations are said to be *covariant.* Later in this section we will give specific examples of covariant physical laws.

[2]An example of a *noninertial frame* is a frame that accelerates in a straight line or rotates with respect to an inertial frame.

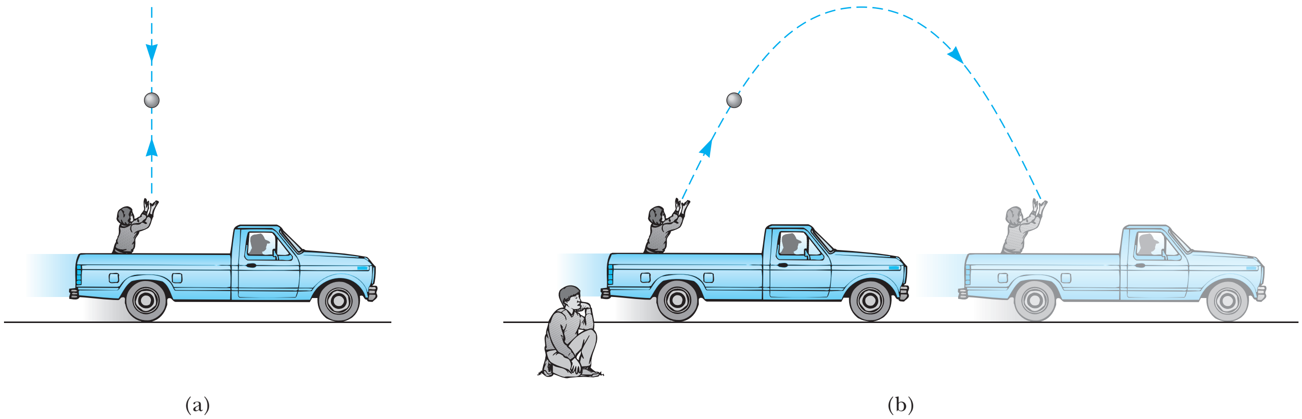

Figure 1.1 The observer in the truck sees the ball move in a vertical path when thrown upward. (b) The Earth observer views the path of the ball as a parabola.

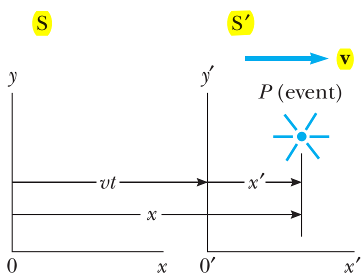

Figure 1.2 An event occurs at a point P. The event is observed by two observers in inertial frames S and S′, in which S′ moves with a velocity **v** relative to S.

In order to show the underlying equivalence of measurements made in different reference frames and hence the equivalence of different frames for doing physics, we need a mathematical formula that systematically relates measurements made in one reference frame to those in another. Such a relation is called a *transformation,* and the one satisfying Newtonian relativity is the so-called *Galilean transformation*, which owes its origin to Galileo. It can be derived as follows.

Consider two inertial systems or frames S and S′, as in Figure 1.2. The frame S′ moves with a constant velocity **v** along the xx' axes, where **v** is measured relative to the frame S. Clocks in S and S′ are synchronized, and the origins of S and S′ coincide at $t = t' = 0$. We assume that a point event, a physical phenomenon such as a lightbulb flash, occurs at the point P. An observer in the system S would describe the event with space–time coordinates (x, y, z, t), whereas an observer in S′ would use (x', y', z', t') to describe the same event. As we can see from Figure 1.2, these coordinates are related by the equations

$$\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned} \tag{1.1}$$

Galilean transformation of coordinates

These equations constitute what is known as a **Galilean transformation of coordinates.** Note that the fourth coordinate, time, is *assumed* to be the same in both inertial frames. That is, *in classical mechanics, all clocks run at the same rate regardless of their velocity*, so that the time at which an event occurs for an observer in S is the same as the time for the same event in S′. Consequently, the time interval between two successive events should be the same for both observers. Although this assumption may seem obvious, it turns out to be *incorrect* when treating situations in which v is comparable to the speed of light. In fact, this point represents one of the most profound differences between Newtonian concepts and the ideas contained in Einstein's theory of relativity.

Exercise 1 Show that although observers in S and S′ measure different coordinates for the ends of a stick at rest in S, they agree on the length of the stick. Assume the stick has end coordinates $x = a$ and $x = a + l$ in S and use the Galilean transformation.

An immediate and important consequence of the invariance of the distance between two points under the Galilean transformation is the invariance of force. For example if $F = \dfrac{kqQ}{(x_2 - x_1)^2}$ gives the electric force between two charges q, Q located at x_1 and x_2 on the x-axis in frame S, F', the force measured in S′, is given by $F' = \dfrac{kqQ}{(x_2' - x_1')^2} = F$ since $x_2' - x_1' = x_2 - x_1$. In fact *any* force would be invariant under the Galilean transformation as long as it involved only the relative positions of interacting particles.

Now suppose two events are separated by a distance dx and a time interval dt as measured by an observer in S. It follows from Equation 1.1 that the corresponding displacement dx' measured by an observer in S′ is given by $dx' = dx - v\,dt$, where dx is the displacement measured by an observer in S. Because $dt = dt'$, we find that

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v$$

or

$$u_x' = u_x - v \qquad (1.2)$$

Galilean addition law for velocities

where u_x and u_x' are the instantaneous velocities of the object relative to S and S′, respectively. This result, which is called the **Galilean addition law for velocities** (or Galilean velocity transformation), is used in everyday observations and is consistent with our intuitive notions of time and space.

To obtain the relation between the accelerations measured by observers in S and S′, we take a derivative of Equation 1.2 with respect to time and use the results that $dt = dt'$ and v is constant:

$$\frac{du_x'}{dt'} = a_x' = a_x \qquad (1.3)$$

Thus observers in different inertial frames measure the same acceleration for an accelerating object. The mathematical terminology is to say that lengths (Δx), time intervals, and accelerations are *invariant* under a Galilean transformation. Example 1.1 points up the distinction between invariant and covariant and shows that **transformation equations, in addition to converting measurements made in one inertial frame to those in another, may be used to show the covariance of physical laws.**

EXAMPLE 1.1 $F_x = ma_x$ Is Covariant Under a Galilean Transformation

Assume that Newton's law $F_x = ma_x$ has been shown to hold by an observer in an inertial frame S. Show that Newton's law also holds for an observer in S′ or is covariant under the Galilean transformation, that is, has the form $F'_x = m'a'_x$. Note that inertial mass is an invariant quantity in Newtonian dynamics.

Solution Starting with the established law $F_x = ma_x$, we use the Galilean transformation $a'_x = a_x$ and the fact that $m' = m$ to obtain $F_x = m'a'_x$. If we now assume that F_x depends only on the relative positions of m and the particles interacting with m, that is, $F_x = f(x_2 - x_1, x_3 - x_1, \ldots)$, then $F_x = F'_x$, because the Δx's are invariant quantities. Thus we find $F'_x = m'a'_x$ and establish the covariance of Newton's second law in this simple case.

Exercise 2 *Conservation of Linear Momentum Is Covariant Under the Galilean Transformation.* Assume that two masses m'_1 and m'_2 are moving in the positive x direction with velocities v'_1 and v'_2 as measured by an observer in S′ before a collision. After the collision, the two masses stick together and move with a velocity v' in S′. Show that if an observer in S′ finds momentum to be conserved, so does an observer in S.

The Speed of Light

It is natural to ask whether the concept of Newtonian relativity and the Galilean addition law for velocities in mechanics also apply to electricity, magnetism, and optics. Recall that Maxwell in the 1860s showed that the speed of light in free space was given by $c = (\mu_0\varepsilon_0)^{-1/2} = 3.00 \times 10^8$ m/s. Physicists of the late 1800s were certain that light waves (like familiar sound and water waves) required a definite medium in which to move, called the *ether*,[3] and that the speed of light was c only with respect to the ether or a frame fixed in the ether called the ether frame. In any other frame moving at speed v relative to the ether frame, the Galilean addition law was expected to hold. Thus, the speed of light in this other frame was expected to be $c - v$ for light traveling in the same direction as the frame, $c + v$ for light traveling opposite to the frame, and in between these two values for light moving in an arbitrary direction with respect to the moving frame.

Because the existence of the ether and a preferred ether frame would show that light was similar to other classical waves (in requiring a medium), considerable importance was attached to establishing the existence of the special ether frame. Because the speed of light is enormous, experiments involving light traveling in media moving at then attainable laboratory speeds had not been capable of detecting small changes of the size of $c \pm v$ prior to the late 1800s. Scientists of the period, realizing that the Earth moved rapidly around

[3]It was proposed by Maxwell that light and other electromagnetic waves were waves in a luminiferous ether, which was present everywhere, even in empty space. In addition to an overblown name, the ether had contradictory properties since it had to have great rigidity to support the high speed of light waves yet had to be tenuous enough to allow planets and other massive objects to pass freely through it, without resistance, as observed.

the Sun at 30 km/s, shrewdly decided to use the Earth itself as the moving frame in an attempt to improve their chances of detecting these small changes in light velocity.

From our point of view of observers fixed on Earth, we may say that we are stationary and that the special ether frame moves past us with speed v. Determining the speed of light under these circumstances is just like determining the speed of an aircraft in a moving air current or wind, and consequently we speak of an "ether wind" blowing through our apparatus fixed to the Earth. If $\mathbf{v}$ is the velocity of the ether relative to the Earth, then the speed of light should have its maximum value, $c + v$, when propagating downwind, as shown in Figure 1.3a. Likewise, the speed of light should have its minimum value, $c - v$, when propagating upwind, as in Figure 1.3b, and an intermediate value, $(c^2 - v^2)^{1/2}$, in the direction perpendicular to the ether wind, as in Figure 1.3c. *If the Sun is assumed to be at rest in the ether,* then the velocity of the ether wind would be equal to the orbital velocity of the Earth around the Sun, which has a magnitude of about 3×10^4 m/s compared to $c = 3 \times 10^8$ m/s. Thus, the change in the speed of light would be about 1 part in 10^4 for measurements in the upwind or downwind directions, and changes of this size should be detectable. However, as we show in the next section, all attempts to detect such changes and establish the existence of the ether proved futile!

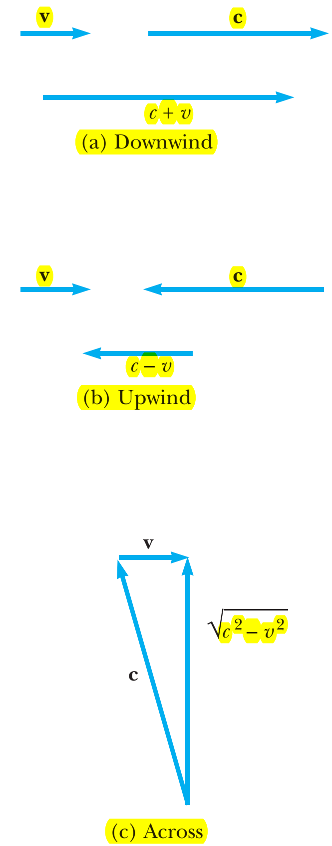

Figure 1.3 If the velocity of the ether wind relative to the Earth is **v,** and **c** is the velocity of light relative to the ether, the speed of light relative to the Earth is (a) $c + v$ in the downwind direction, (b) $c - v$ in the upwind direction, and (c) $(c^2 - v^2)^{1/2}$ in the direction perpendicular to the wind.

1.3 THE MICHELSON–MORLEY EXPERIMENT

The famous experiment designed to detect small changes in the speed of light with motion of an observer through the ether was performed in 1887 by American physicist Albert A. Michelson (1852–1931) and the American chemist Edward W. Morley (1838–1923).[4] We should state at the outset that the outcome of the experiment was *negative,* thus contradicting the ether hypothesis. The highly accurate experimental tool perfected by these pioneers to measure small changes in light speed was the Michelson interferometer, shown in Figure 1.4. One of the arms of the interferometer was aligned along the direction of the motion of the Earth through the ether. The Earth moving through the ether would be equivalent to the ether flowing past the Earth in the opposite direction with speed v, as shown in Figure 1.4. This ether wind blowing in the opposite direction should cause the speed of light measured in the Earth's frame of reference to be $c - v$ as it approaches the mirror M_2 in Figure 1.4 and $c + v$ after reflection. The speed v is the speed of the Earth through space, and hence the speed of the ether wind, and c is the speed of light in the ether frame. The two beams of light reflected from M_1 and M_2 would recombine, and an interference pattern consisting of alternating dark and bright bands, or fringes, would be formed.

During the experiment, the interference pattern was observed while the interferometer was rotated through an angle of 90°. This rotation would change the speed of the ether wind along the direction of the arms of the interferometer. The effect of this rotation should have been to cause the fringe pattern to shift slightly but measurably. Measurements failed to show any change in the

[4] A. A. Michelson and E. W. Morley, *Am. J. Sci.* 134:333, 1887.

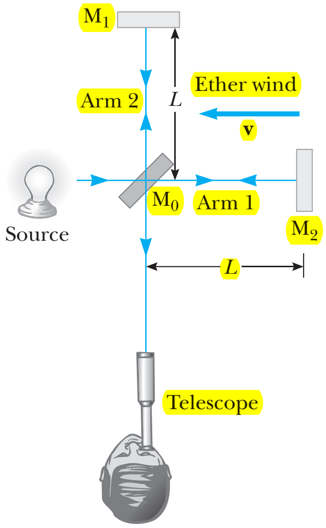

Figure 1.4 Diagram of the Michelson interferometer. According to the ether wind concept, the speed of light should be $c - v$ as the beam approaches mirror M_2 and $c + v$ after reflection.

interference pattern! The Michelson–Morley experiment was repeated by other researchers under various conditions and at different times of the year when the ether wind was expected to have changed direction and magnitude, but the results were always the same: *No fringe shift of the magnitude required was ever observed.*[5]

The negative results of the Michelson–Morley experiment not only meant that the speed of light does not depend on the direction of light propagation but also contradicted the ether hypothesis. The negative results also meant that it was impossible to measure the absolute velocity of the Earth with respect to the ether frame. As we shall see in the next section, Einstein's postulates compactly explain these and a host of other perplexing questions, relegating the idea of the ether to the ash heap of history. Light is now understood to be a phenomenon that *requires no medium for its propagation*. As a result, the idea of an ether in which these waves could travel became unnecessary.

Details of the Michelson–Morley Experiment

To understand the outcome of the Michelson–Morley experiment, let us assume that the interferometer shown in Figure 1.4 has two arms of equal length L. First consider the beam traveling parallel to the direction of the ether wind, which is taken to be horizontal in Figure 1.4. According to Newtonian mechanics, as the beam moves to the right, its speed is reduced by the wind and its speed with respect to the Earth is $c - v$. On its return journey, as the light beam moves to the left downwind, its speed with respect to the Earth is $c + v$. Thus, the time of travel to the right is $L/(c - v)$, and the time of travel to the left is $L/(c + v)$. The total time of travel for the round-trip along the horizontal path is

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \frac{2Lc}{c^2 - v^2} = \frac{2L}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1}$$

Now consider the light beam traveling perpendicular to the wind, as shown in Figure 1.4. Because the speed of the beam relative to the Earth is $(c^2 - v^2)^{1/2}$ in this case (see Fig. 1.3c), the time of travel for each half of this trip is $L/(c^2 - v^2)^{1/2}$, and the total time of travel for the round-trip is

$$t_2 = \frac{2L}{(c^2 - v^2)^{1/2}} = \frac{2L}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

Thus, the time difference between the light beam traveling horizontally and the beam traveling vertically is

$$\Delta t = t_1 - t_2 = \frac{2L}{c}\left[\left(1 - \frac{v^2}{c^2}\right)^{-1} - \left(1 - \frac{v^2}{c^2}\right)^{-1/2}\right]$$

[5]From an Earth observer's point of view, changes in the Earth's speed and direction in the course of a year are viewed as ether wind shifts. In fact, even if the speed of the Earth with respect to the ether were zero at some point in the Earth's orbit, six months later the speed of the Earth would be 60 km/s with respect to the ether, and one should find a clear fringe shift. None has ever been observed, however.

Because $v^2/c^2 \ll 1$, this expression can be simplified by using the following binomial expansion after dropping all terms higher than second order:

$$(1 - x)^n \approx 1 - nx \quad (\text{for } x \ll 1)$$

In our case, $x = v^2/c^2$, and we find

$$\Delta t = t_1 - t_2 \approx \frac{Lv^2}{c^3} \tag{1.4}$$

The two light beams start out in phase and return to form an interference pattern. Let us assume that the interferometer is adjusted for parallel fringes and that a telescope is focused on one of these fringes. The time difference between the two light beams gives rise to a phase difference between the beams, producing the interference fringe pattern when they combine at the position of the telescope. A difference in the pattern (Fig. 1.6) should be detected by rotating the interferometer through 90° in a horizontal plane, such that the two beams exchange roles. This results in a net time difference of twice that given by Equation 1.4. The path difference corresponding to this time difference is

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$$\Delta d = c(2\Delta t) = \frac{2Lv^2}{c^2}$$

The corresponding fringe shift is equal to this path difference divided by the wavelength of light, λ, because a change in path of 1 wavelength corresponds to a shift of 1 fringe.

$$\text{Shift} = \frac{2Lv^2}{\lambda c^2} \tag{1.5}$$

In the experiments by Michelson and Morley, each light beam was reflected by mirrors many times to give an increased effective path length L of about 11 m. Using this value, and taking v to be equal to 3×10^4 m/s, the speed of the Earth about the Sun, gives a path difference of

$$\Delta d = \frac{2(11\text{ m})(3 \times 10^4\text{ m/s})^2}{(3 \times 10^8\text{ m/s})^2} = 2.2 \times 10^{-7}\text{ m}$$

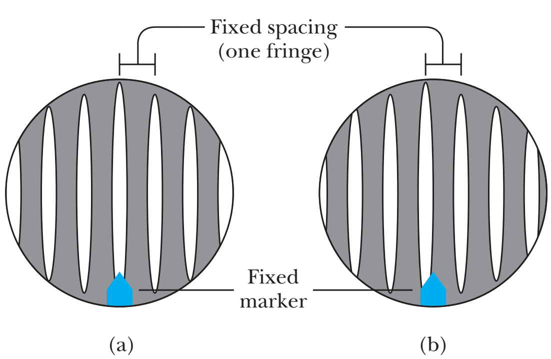

Figure 1.6 Interference fringe schematic showing (a) fringes before rotation and (b) expected fringe shift after a rotation of the interferometer by 90°.

This extra distance of travel should produce a noticeable shift in the fringe pattern. Specifically, using light of wavelength 500 nm, we find a fringe shift for rotation through 90° of

$$\text{Shift} = \frac{\Delta d}{\lambda} = \frac{2.2 \times 10^{-7}\ \text{m}}{5.0 \times 10^{-7}\ \text{m}} \approx 0.40$$

The precision instrument designed by Michelson and Morley had the capability of detecting a shift in the fringe pattern as small as 0.01 fringe. However, *they detected no shift in the fringe pattern.* Since then, the experiment has been repeated many times by various scientists under various conditions, and no fringe shift has ever been detected. Thus, it was concluded that one cannot detect the motion of the Earth with respect to the ether.

Many efforts were made to explain the null results of the Michelson–Morley experiment and to save the ether concept and the Galilean addition law for the velocity of light. Because all these proposals have been shown to be wrong, we consider them no further here and turn instead to an auspicious proposal made by George F. Fitzgerald and Hendrik A. Lorentz. In the 1890s, Fitzgerald and Lorentz tried to explain the null results by making the following ad hoc assumption. They proposed that the length of an object moving at speed v would contract along the direction of travel by a factor of $\sqrt{1 - v^2/c^2}$. The net result of this contraction would be a change in length of one of the arms of the interferometer such that no path difference would occur as the interferometer was rotated.

Never in the history of physics were such valiant efforts devoted to trying to explain the absence of an expected result as those directed at the Michelson–Morley experiment. The difficulties raised by this null result were tremendous, not only implying that light waves were a new kind of wave propagating without a medium but that the Galilean transformations were flawed for inertial frames moving at high relative speeds. The stage was set for Albert Einstein, who solved these problems in 1905 with his special theory of relativity.

1.4 POSTULATES OF SPECIAL RELATIVITY

In the previous section we noted the impossibility of measuring the speed of the ether with respect to the Earth and the failure of the Galilean velocity transformation in the case of light. In 1905, Albert Einstein (Fig. 1.7) proposed a theory that boldly removed these difficulties and at the same time completely altered our notion of space and time.[6] Einstein based his special theory of relativity on two postulates.

Postulates of special relativity

1. **The Principle of Relativity:** All the laws of physics have the same form in all inertial reference frames.
2. **The Constancy of the Speed of Light:** The speed of light in vacuum has the same value, $c = 3.00 \times 10^8$ m/s, in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

[6] A. Einstein, "On the Electrodynamics of Moving Bodies," *Ann. Physik* 17:891, 1905. For an English translation of this article and other publications by Einstein, see the book by H. Lorentz, A. Einstein, H. Minkowski, and H. Weyl, *The Principle of Relativity,* Dover, 1958.

B I O G R A P H Y

ALBERT EINSTEIN
(1879–1955)

Albert Einstein, one of the greatest physicists of all time, was born in Ulm, Germany. As a child, Einstein was very unhappy with the discipline of German schools and completed his early education in Switzerland at age 16. Because he was unable to obtain an academic position following graduation from the Swiss Federal Polytechnic School in 1901, he accepted a job at the Swiss Patent Office in Berne. During his spare time, he continued his studies in theoretical physics. In 1905, at the age of 26, he published four scientific papers that revolutionized physics. One of these papers, which won him the Nobel prize in 1921, dealt with the photoelectric effect. Another was concerned with Brownian motion, the irregular motion of small particles suspended in a liquid. The remaining two papers were concerned with what is now considered his most important contribution of all, the special theory of relativity. In 1915, Einstein published his work on the general theory of relativity, which relates gravity to the structure of space and time. One of the remarkable predictions of the theory is that strong gravitational forces in the vicinity of very massive objects cause light beams to deviate from straight-line paths. This and other predictions of the general theory of relativity have been experimentally verified (see the essay on our companion Web site by Clifford Will).

Einstein made many other important contributions to the development of modern physics, including the concept of the light quantum and the idea of stimulated emission of radiation, which led to the invention of the laser 40 years later. However, throughout his life, he rejected the probabilistic interpretation of quantum mechanics when describing events on the atomic scale in favor of a deterministic view. He is quoted as saying, "God does not play dice with the universe." This comment is reputed to have been answered by Niels Bohr, one of the founders of quantum mechanics, with "Don't tell God what to do!"

In 1933, Einstein left Germany (by then under Nazis control) and spent his remaining years at the Institute for Advanced Study in Princeton, New Jersey. He devoted most of his later years to an unsuccessful search for a unified theory of gravity and electromagnetism.

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The first postulate asserts that *all* the laws of physics, those dealing with electricity and magnetism, optics, thermodynamics, mechanics, and so on, will have the same mathematical form or be covariant in all coordinate frames moving with constant velocity relative to one another. This postulate is a sweeping generalization of Newton's principle of relativity, which refers only to the laws of mechanics. From an experimental point of view, Einstein's principle of relativity means that no experiment of any type can establish an absolute rest frame, and that all inertial reference frames are experimentally indistinguishable.

Note that postulate 2, the principle of the constancy of the speed of light, is consistent with postulate 1: If the speed of light was not the same in all inertial frames but was c in only one, it would be possible to distinguish between inertial frames, and one could identify a preferred, absolute frame in contradiction to postulate 1. Postulate 2 also does away with the problem of measuring the speed of the ether by essentially denying the existence of the ether and boldly asserting that light always moves with speed c with respect to any inertial observer. Postulate 2 was a brilliant theoretical insight on Einstein's part in 1905 and has since been directly confirmed experimentally in many ways. Perhaps the most direct demonstration involved measuring the speed of very high frequency electromagnetic waves (gamma rays) emitted by unstable particles (neutral pions) traveling at 99.975% of the speed of light with respect to the laboratory. The measured gamma ray speed relative to the laboratory agreed in this case to five significant figures with the speed of light in empty space.

The Michelson–Morley experiment was performed before Einstein published his work on relativity, and it is not clear that Einstein was aware of the details of the experiment. Nonetheless, the null result of the experiment can be readily understood within the framework of Einstein's theory. According to his principle of relativity, the premises of the Michelson–Morley experiment were incorrect. In the process of trying to explain the expected results, we stated that when light traveled against the ether wind its speed was $c - v$, in accordance with the Galilean addition law for velocities. However, if the state of motion of the observer or of the source has no influence on the value found for the speed of light, one will always measure the value to be c. Likewise, the light makes the return trip after reflection from the mirror at a speed of c, and not with the speed $c + v$. Thus, the motion of the Earth should not influence the fringe pattern observed in the Michelson–Morley experiment, and a null result should be expected.

Perhaps at this point you have rightly concluded that the Galilean velocity and coordinate transformations are incorrect; that is, the Galilean transformations do not keep all the laws of physics in the same form for different inertial frames. The correct coordinate and time transformations that preserve the covariant form of all physical laws in two coordinate systems moving uniformly with respect to each other are called *Lorentz transformations.* These are derived in Section 1.6. Although the Galilean transformation preserves the form of Newton's laws in two frames moving uniformly with respect to each other, Newton's laws of mechanics are limited laws that are valid only for low speeds. In general, Newton's laws must be replaced by Einstein's relativistic laws of mechanics, which hold for all speeds and are invariant, as are all physical laws, under the Lorentz transformations.

1.5 CONSEQUENCES OF SPECIAL RELATIVITY

Almost everyone who has dabbled even superficially with science is aware of some of the startling predictions that arise because of Einstein's approach to relative motion. As we examine some of the consequences of relativity in this section, we shall find that they conflict with our basic notions of space and time. We restrict our discussion to the concepts of length, time, and simultaneity, which are quite different in relativistic mechanics and Newtonian mechanics. For example, we will find that *the distance between two points and the time interval between two events depend on the frame of reference in which they are measured.* That is, *there is no such thing as absolute length or absolute time in relativity.* Furthermore, *events at different locations that occur simultaneously in one frame are not simultaneous in another frame moving uniformly past the first.*

Before we discuss the consequences of special relativity, we must first understand how an observer in an inertial reference frame describes an event. We define an event as an occurrence described by three space coordinates and one time coordinate. In general, different observers in different inertial frames would describe the same event with different spacetime coordinates.

The reference frame used to describe an event consists of a coordinate grid and a set of clocks situated at the grid intersections, as shown in Figure 1.8 in two dimensions. It is necessary that the clocks be synchronized. This can be accomplished in many ways with the help of light signals. For example, suppose an observer at the origin with a master clock sends out a pulse of light at $t = 0$. The light pulse takes a time r/c to reach a second clock, situated a distance r from the origin. Hence, the second clock will be synchronized with the clock at the origin if the second clock reads a time r/c at the instant the pulse reaches it. This procedure of synchronization assumes that the speed of light has the same value in all directions and in all inertial frames. Furthermore, the procedure concerns an event recorded by an observer in a specific inertial reference frame. Clocks in other inertial frames can be synchronized in a similar manner. An observer in some other inertial frame would assign different spacetime coordinates to events, using another coordinate grid with another array of clocks.

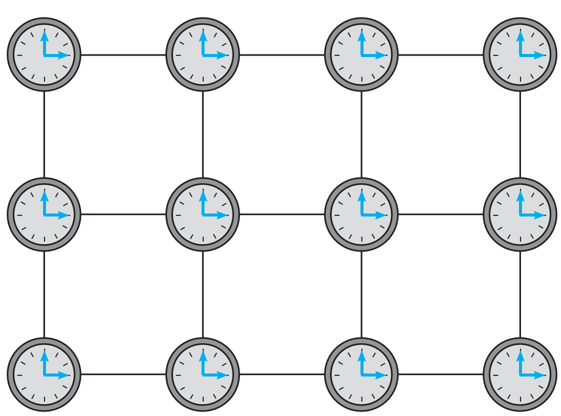

Figure 1.8 In relativity, we use a reference frame consisting of a coordinate grid and a set of synchronized clocks.

Simultaneity and the Relativity of Time

A basic premise of Newtonian mechanics is that a universal time scale exists that is the same for all observers. In fact, Newton wrote that "Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external." Thus, Newton and his followers simply took simultaneity for granted. In his special theory of relativity, Einstein abandoned this assumption. According to Einstein, *a time interval measurement depends on the reference frame in which the measurement is made.*

Einstein devised the following thought experiment to illustrate this point. A boxcar moves with uniform velocity, and two lightning bolts strike the ends of the boxcar, as in Figure 1.9a, leaving marks on the boxcar and ground. The marks left on the boxcar are labeled A' and B'; those on the ground are labeled A and B. An observer at O' moving with the boxcar is midway between A' and B', and a ground observer at O is midway between A and B. The events recorded by the observers are the light signals from the lightning bolts.

The two light signals reach the observer at O at the same time, as indicated in Figure 1.9b. This observer realizes that the light signals have traveled at the same speed over equal distances. Thus, observer O concludes that the events at A and B occurred simultaneously. Now consider the same events as viewed by the observer on the boxcar at O'. By the time the light has reached observer O, observer O' has moved as indicated in Figure 1.9b. Thus, the light signal from B' has already swept past O', but the light from A' has not yet reached O'. According to Einstein, *observer O' must find that light travels at the same speed as that measured by observer O.* Therefore, observer O' concludes that the lightning struck the front of the boxcar before it struck the back. This thought experiment clearly demonstrates that the two events, which appear to O to be simultaneous, do not appear to O' to be simultaneous. In other words,

> Two events that are simultaneous in one frame are in general not simultaneous in a second frame moving with respect to the first. That is, simultaneity is not an absolute concept, but one that depends on the state of motion of the observer.

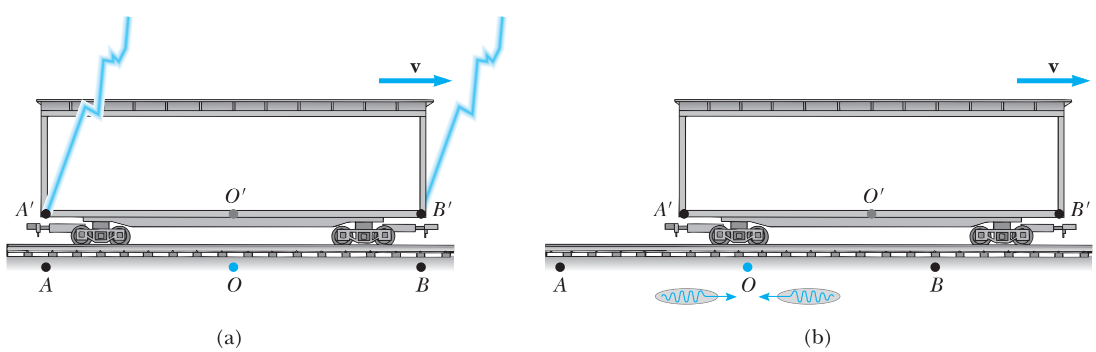

Figure 1.9 Two lightning bolts strike the ends of a moving boxcar. (a) The events appear to be simultaneous to the stationary observer at O, who is midway between A and B. (b) The events do not appear to be simultaneous to the observer at O', who claims that the front of the train is struck *before* the rear.

At this point, you might wonder which observer is right concerning the two events. The answer is that *both are correct,* because the principle of relativity states that *there is no preferred inertial frame of reference.* Although the two observers reach different conclusions, both are correct in their own reference frame because the concept of simultaneity is not absolute. This, in fact, is the central point of relativity—any uniformly moving frame of reference can be used to describe events and do physics. However, observers in different inertial frames will always measure different time intervals with their clocks and different distances with their meter sticks. Nevertheless, they will both agree on the forms of the laws of physics in their respective frames, because these laws must be the same for all observers in uniform motion. It is the alteration of time and space that allows the laws of physics (including Maxwell's equations) to be the same for all observers in uniform motion.

Time Dilation

The fact that observers in different inertial frames always measure different time intervals between a pair of events can be illustrated in another way by considering a vehicle moving to the right with a speed v, as in Figure 1.10a. A mirror is fixed to the ceiling of the vehicle, and observer O', at rest in this system, holds a laser a distance d below the mirror. At some instant the laser emits a pulse of light directed toward the mirror (event 1), and at some later time, after reflecting from the mirror, the pulse arrives back at the laser (event 2). Observer O' carries a clock, C', which she uses to measure the time interval $\Delta t'$ between these two events. Because the light pulse has the speed c, the time it takes to travel from O' to the mirror and back can be found from the definition of speed:

$$\Delta t' = \frac{\text{distance traveled}}{\text{speed of light}} = \frac{2d}{c} \tag{1.6}$$

This time interval $\Delta t'$—measured by O', who, remember, is at rest in the moving vehicle—requires only a *single* clock, C', in this reference frame.

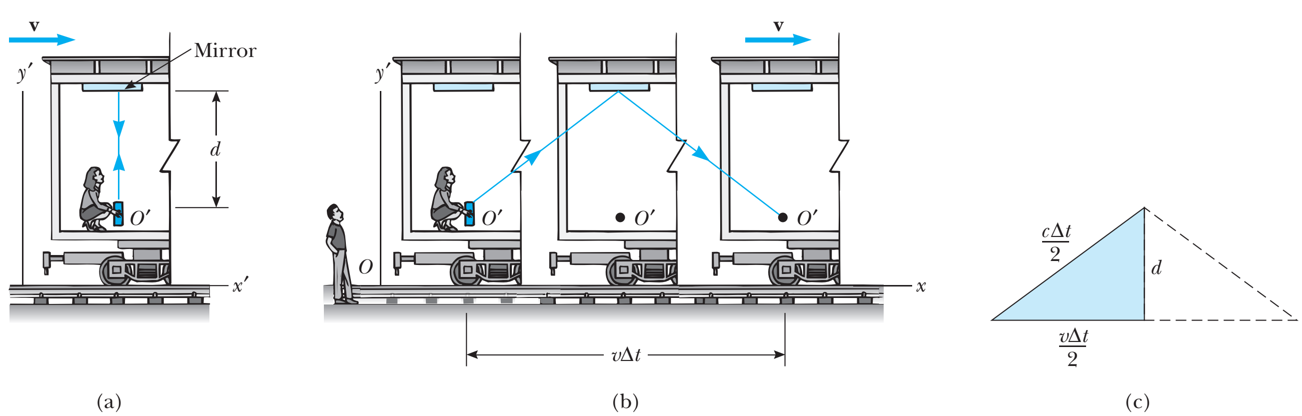

Figure 1.10 (a) A mirror is fixed to a moving vehicle, and a light pulse leaves O' at rest in the vehicle. (b) Relative to a stationary observer on Earth, the mirror and O' move with a speed v. Note that the distance the pulse travels measured by the stationary observer on Earth is greater than $2d$. (c) The right triangle for calculating the relationship between Δt and $\Delta t'$.

Now consider the same set of events as viewed by observer O in a second frame (Fig. 1.10b). According to this observer, the mirror and laser are moving to the right with a speed v, and as a result, the sequence of events appears different to this observer. By the time the light from the laser reaches the mirror, the mirror has moved to the right a distance $v\Delta t/2$, where Δt is the time interval required for the light pulse to travel from O' to the mirror and back as measured by O. In other words, O concludes that, because of the motion of the vehicle, if the light is to hit the mirror, it must leave the laser at an angle with respect to the vertical direction. Comparing Figures 1.10a and 1.10b, we see that the light must travel farther in (b) than in (a). (Note that neither observer "knows" that he or she is moving. Each is at rest in his or her own inertial frame.)

According to the second postulate of special relativity, both observers must measure c for the speed of light. Because the light travels farther according to O, it follows that the time interval Δt measured by O is longer than the time interval $\Delta t'$ measured by O'. To obtain a relationship between Δt and $\Delta t'$, it is convenient to use the right triangle shown in Figure 1.10c. The Pythagorean theorem gives

$$\left(\frac{c\Delta t}{2}\right)^2 = \left(\frac{v\Delta t}{2}\right)^2 + d^2$$

Solving for Δt gives

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1 - v^2/c^2}} \tag{1.7}$$

Because $\Delta t' = 2d/c$, we can express Equation 1.7 as

Time dilation

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - (v^2/c^2)}} = \gamma \Delta t' \tag{1.8}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$. Because γ is always greater than unity, this result says that the time interval Δt measured by the observer moving with respect to the clock is *longer* than the time interval $\Delta t'$ measured by the observer at rest with respect to the clock. This effect is known as **time dilation.**

A moving clock runs slower

The time interval $\Delta t'$ in Equation 1.8 is called the proper time. In general, **proper time,** denoted Δt_p, **is defined as the time interval between two events as measured by an observer who sees the events occur at the same point in space.** In our case, observer O' measures the proper time. That is, **proper time is always the time measured by an observer moving along with the clock.** As an aid in solving problems it is convenient to express Equation 1.8 in terms of the proper time interval, Δt_p, as

$$\Delta t = \gamma \Delta t_p \tag{1.9}$$

Because the time between ticks of a moving clock, $\gamma(2d/c)$, is observed to be longer than the time between ticks of an identical clock at rest, $2d/c$, one commonly says, *"A moving clock runs slower than a clock at rest by a factor of* γ." This is true for ordinary mechanical clocks as well as for the light clock just described. In fact, we can generalize these results by stating that *all physical processes,* including chemical reactions and biological processes, slow down when observed from a reference frame in which they are moving. For example, the heartbeat of an astronaut moving through space would keep time with a clock inside the spaceship, but both the astronaut's clock and her heartbeat appear slow to an observer, with another clock, in any other reference frame. The astronaut would not have any sensation of life slowing down in her frame.

Time dilation is a very real phenomenon that has been verified by various experiments. For example, muons are unstable elementary particles that have a charge equal to that of an electron and a mass 207 times that of the electron. Muons are naturally produced by the collision of cosmic radiation with atoms at a height of several thousand meters above the surface of the Earth. Muons have a lifetime of only 2.2 μs when measured in a reference frame at rest with respect to them. If we take 2.2 μs (proper time) as the average lifetime of a muon and assume that its speed is close to the speed of light, we would find that these particles could travel a distance of about 650 m before they decayed. Hence, they could not reach the Earth from the upper atmosphere where they are produced. However, experiments show that a large number of muons *do* reach the Earth. The phenomenon of time dilation explains this effect (see Fig. 1.11a). Relative to an observer on Earth, the muons have a lifetime equal to $\gamma\tau$, where $\tau = 2.2\ \mu$s is the lifetime in a frame of reference traveling with the muons. For example, for $v = 0.99c$, $\gamma \approx 7.1$ and $\gamma\tau \approx 16\ \mu$s. Hence, the average distance traveled as measured by an observer on Earth is $\gamma v\tau \approx 4700$ m, as indicated in Figure 1.11b.

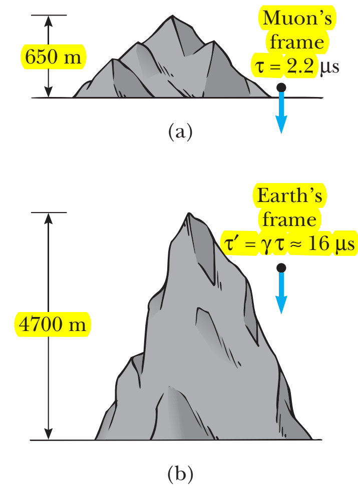

Figure 1.11 (a) Muons traveling with a speed of $0.99c$ travel only about 650 m as measured in the muons' reference frame, where their lifetime is about 2.2 μs. (b) The muons travel about 4700 m as measured by an observer on Earth. Because of time dilation, the muons' lifetime is longer as measured by the Earth observer.

In 1976, experiments with muons were conducted at the laboratory of the European Council for Nuclear Research (CERN) in Geneva. Muons were injected into a large storage ring, reaching speeds of about $0.9994c$. Electrons produced by the decaying muons were detected by counters around the ring, enabling scientists to measure the decay rate, and hence the lifetime, of the muons. The lifetime of the moving muons was measured to be about 30 times as long as that of the stationary muon (see Fig. 1.12), in agreement with the prediction of relativity to within two parts in a thousand.

It is quite interesting that time dilation can be observed directly by comparing high-precision atomic clocks, one carried aboard a jet, the other

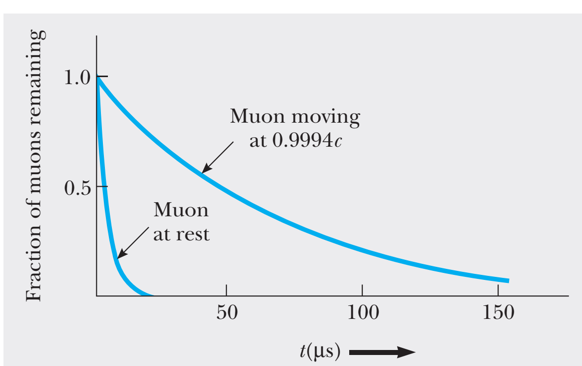

Figure 1.12 Decay curves for muons traveling at a speed of $0.9994c$ and for muons at rest.

remaining in a laboratory on Earth. The actual experiment involved the use of very stable cesium beam atomic clocks.[7] Time intervals measured with four such clocks in jet flight were compared with time intervals measured by reference atomic clocks located at the U.S. Naval Observatory. To compare these results with the theory, many factors had to be considered, including periods of acceleration and deceleration relative to the Earth, variations in direction of travel, and the weaker gravitational field experienced by the flying clocks compared with the Earth-based clocks. The results were in good agreement with the predictions of the special theory of relativity and can be completely explained in terms of the relative motion between the Earth and the jet aircraft.

EXAMPLE 1.2 What Is the Period of the Pendulum?

The period of a pendulum is measured to be 3.0 s in the rest frame of the pendulum. What is the period of the pendulum when measured by an observer moving at a speed of $0.95c$ with respect to the pendulum?

Solution In this case, the proper time is equal to 3.0 s. From the point of view of the observer, the pendulum is moving at $0.95c$ past her. Hence the pendulum is an example of a moving clock. Because a moving clock runs slower than a stationary clock by γ, Equation 1.8 gives

$$T = \gamma T' = \frac{1}{\sqrt{1 - (0.95c)^2/c^2}}\, 3.0 \text{ s}$$

$$T = (3.2)(3.0 \text{ s}) = 9.6 \text{ s}$$

That is, a moving pendulum slows down or takes longer to complete one period.

Exercise 3 If the speed of the observer is increased by 5.0%, what is the period of the pendulum when measured by this observer?

Answer 43 s. Note that the 5.0% increase in speed causes more than a 300% increase in the dilated time.

Length Contraction

We have seen that measured time intervals are not absolute, that is, the time interval between two events depends on the frame of reference in which it is measured. Likewise, the measured distance between two points depends on the frame of reference. **The proper length of an object is defined as the length of the object measured by someone who is at rest with respect to the object.** You should note that proper length is defined similarly to proper time, in that proper time is the time between ticks of a clock measured by an observer who is at rest with respect to the clock. The length of an object measured by someone in a reference frame that is moving relative to the object is always less than the proper length. This effect is known as **length contraction.**

To understand length contraction quantitatively, consider a spaceship traveling with a speed v from one star to another and two observers, one on Earth

[7] J. C. Hafele and R. E. Keating, "Around the World Atomic Clocks: Relativistic Time Gains Observed," *Science,* July 14, 1972, p. 168.

and the other in the spaceship. The observer at rest on Earth (and also assumed to be at rest with respect to the two stars) measures the distance between the stars to be L_p, where L_p is the proper length. According to this observer, the time it takes the spaceship to complete the voyage is $\Delta t = L_p/v$. What does an observer in the moving spaceship measure for the distance between the stars? Because of time dilation, the space traveler measures a smaller time of travel: $\Delta t' = \Delta t/\gamma$. The space traveler claims to be at rest and sees the destination star as moving toward the spaceship with speed v. Because the space traveler reaches the star in the shorter time $\Delta t'$, he or she concludes that the distance, L, between the stars is shorter than L_p. This distance measured by the space traveler is given by

$$L = v\Delta t' = v\frac{\Delta t}{\gamma}$$

Because $L_p = v\Delta t$, we see that $L = L_p/\gamma$ or

$$L = L_p\left(1 - \frac{v^2}{c^2}\right)^{1/2} \tag{1.10}$$

Length contraction

where $(1 - v^2/c^2)^{1/2}$ is a factor less than 1. This result may be interpreted as follows:

> If an object has a proper length L_p when it is measured by an observer at rest with respect to the object, when it moves with speed v in a direction parallel to its length, its length L is measured to be shorter according to $L = L_p\left(1 - \frac{v^2}{c^2}\right)^{1/2}$.

Note that the length contraction takes place only along the direction of motion. For example, suppose a stick moves past a stationary Earth observer with a speed v, as in Figure 1.13b. The length of the stick as measured by an observer in the frame attached to it is the proper length L_p, as illustrated in Figure 1.13a. The length of the stick, L, as measured by the Earth observer is shorter than L_p by the factor $(1 - v^2/c^2)^{1/2}$. Note that length contraction is a symmetric effect: If the stick were at rest on Earth, an observer in a frame moving past the earth at speed v would also measure its length to be shorter by the same factor $(1 - v^2/c^2)^{1/2}$.

As we mentioned earlier, one of the basic tenets of relativity is that all inertial frames are equivalent for analyzing an experiment. Let us return to the example of the decaying muons moving at speeds close to the speed of light to see an example of this. An observer in the muon's reference frame would measure the proper lifetime, whereas an Earth-based observer measures the proper height of the mountain in Figure 1.11. In the muon's reference frame, there is no time dilation, but the distance of travel is observed to be shorter when measured from this frame. Likewise, in the Earth observer's reference frame, there is time dilation, but the distance of travel is measured to be the proper height of the mountain. Thus, when calculations on the muon are performed in both frames, one sees the effect of "offsetting penalties," and the outcome of the experiment is the same!

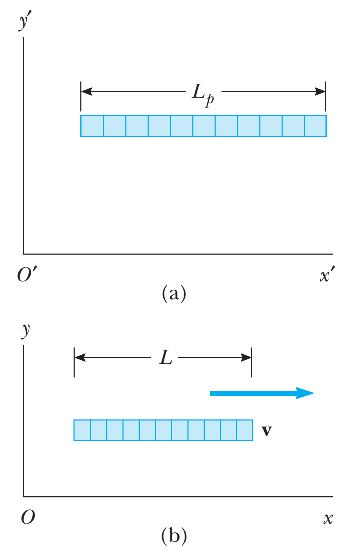

Figure 1.13 A stick moves to the right with a speed v. (a) The stick as viewed in a frame attached to it. (b) The stick as seen by an observer who sees it move past her at v. Any inertial observer finds that the length of a meter stick moving past her with speed v is less than the length of a stationary stick by a factor of $(1 - v^2/c^2)^{1/2}$.

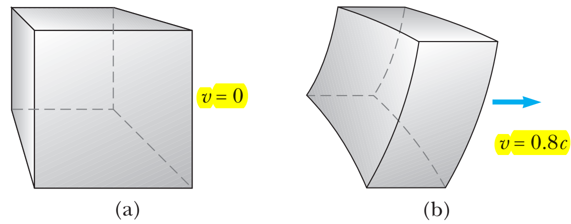

Figure 1.14 Computer-simulated photographs of a box (a) at rest relative to the camera and (b) moving at a speed $v = 0.8c$ relative to the camera.

Note that proper length and proper time are measured in *different* reference frames.

If an object in the shape of a box passing by could be photographed, its image would show length contraction, but its shape would also be distorted. This is illustrated in the computer-simulated drawings shown in Figure 1.14 for a box moving past an observer with a speed $v = 0.8c$. When the shutter of the camera is opened, it records the shape of the object at a given instant of time. Because light from different parts of the object must arrive at the shutter at the same time (when the photograph is taken), light from more distant parts of the object must start its journey earlier than light from closer parts. Hence, the photograph records different parts of the object at different times. This results in a highly distorted image, which shows horizontal length contraction, vertical curvature, and image rotation.

EXAMPLE 1.3 The Contraction of a Spaceship

A spaceship is measured to be 100 m long while it is at rest with respect to an observer. If this spaceship now flies by the observer with a speed of $0.99c$, what length will the observer find for the spaceship?

Solution The proper length of the ship is 100 m. From Equation 1.10, the length measured as the spaceship flies by is

$$L = L_p\sqrt{1 - \frac{v^2}{c^2}} = (100 \text{ m})\sqrt{1 - \frac{(0.99c)^2}{c^2}} = 14 \text{ m}$$

Exercise 4 If the ship moves past the observer at $0.01000c$, what length will the observer measure?

Answer 99.99 m.

EXAMPLE 1.4 How High Is the Spaceship?

An observer on Earth sees a spaceship at an altitude of 435 m moving downward toward the Earth at $0.970c$. What is the altitude of the spaceship as measured by an observer in the spaceship?

Solution The proper length here is the Earth–ship separation as seen by the Earth-based observer, or 435 m. The moving observer in the ship finds this separation (the altitude) to be

$$L = L_p\sqrt{1 - \frac{v^2}{c^2}} = (435 \text{ m})\sqrt{1 - \frac{(0.970c)^2}{c^2}}$$

$$= 106 \text{ m}$$

EXAMPLE 1.5 The Triangular Spaceship

A spaceship in the form of a triangle flies by an observer at $0.950c$. When the ship is measured by an observer at rest with respect to the ship (Fig. 1.15a), the distances x and y are found to be 50.0 m and 25.0 m, respectively. What is the shape of the ship as seen by an observer who sees the ship in motion along the direction shown in Figure 1.15b?

Solution The observer sees the horizontal length of the ship to be contracted to a length of

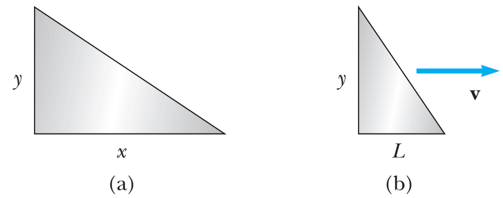

Figure 1.15 (Example 1.5) (a) When the spaceship is at rest, its shape is as shown. (b) The spaceship appears to look like this when it moves to the right with a speed v. Note that only its x dimension is contracted in this case.

$$L = L_p\sqrt{1 - \frac{v^2}{c^2}}$$

$$= (50.0 \text{ m})\sqrt{1 - \frac{(0.950c)^2}{c^2}} = 15.6 \text{ m}$$

The 25-m vertical height is unchanged because it is perpendicular to the direction of relative motion between the observer and the spaceship. Figure 1.15b represents the shape of the spaceship as seen by the observer who sees the ship in motion.

THE TWINS PARADOX

O P T I O N A L

If we placed a living organism in a box . . . one could arrange that the organism, after an arbitrary lengthy flight, could be returned to its original spot in a scarcely altered condition, while corresponding organisms which had remained in their original positions had long since given way to new generations. **(Einstein's original statement of the twins paradox in 1911)**

An intriguing consequence of time dilation is the so-called clock or twins paradox. Consider an experiment involving a set of identical 20-year-old twins named Speedo and Goslo. The twins carry with them identical clocks that have been synchronized. Speedo, the more adventuresome of the two, sets out on an epic journey to planet X, 10 lightyears from Earth. (Note that 1 lightyear (ly) is the distance light travels through free space in 1 year.) Furthermore, his spaceship is capable of a speed of $0.500c$ relative to the inertial frame of his twin brother. After reaching planet X, Speedo becomes homesick and impetuously sets out on a return trip to Earth at the same high speed of the outbound journey. On his return, Speedo is shocked to discover that many things have changed during his absence. To Speedo, the most significant change is that his twin brother Goslo *has aged more than he* and is now 60 years of age. Speedo, on the other hand, has aged by only 34.6 years.

At this point, it is fair to raise the following question—Which twin is the traveler and which twin would really be the younger of the two? If motion is relative, the twins are in a symmetric situation and either's point of view is equally valid. From Speedo's perspective, it is he who is at rest while Goslo is on a high-speed space journey. To Speedo, it is Goslo and the Earth that have raced away on a 17.3-year journey and then headed back for another 17.3 years. This leads to the paradox: Which twin will have developed the signs of excess aging?

To resolve this apparent paradox, recall that special relativity deals with inertial frames of reference moving with respect to one another at *uniform speed.* However, the trip situation is not symmetric. Speedo, the space traveler, must experience acceleration during his journey. As a result, his state of motion is not always uniform, and consequently Speedo is not in an inertial frame. He cannot regard himself to always be at rest and Goslo to be in uniform motion. Hence Speedo cannot apply simple time dilation to Goslo's motion, because to do so would be an incorrect application of special relativity. Therefore there is no paradox and Speedo will really be the younger twin at the end of the trip.

The conclusion that Speedo is not in a *single* inertial frame is inescapable. We may diminish the length of time needed to accelerate and decelerate Speedo's spaceship to an insignificant interval by using very large and expensive rockets and claim that he spends all but a negligible amount of time coasting to planet X at $0.500c$ in an inertial frame. However, to return to Earth, Speedo must slow down, reverse his motion, and return in a different inertial frame, one which is moving uniformly toward the Earth. At the very best, Speedo is in *two* different inertial frames. The important point is that even when we idealize Speedo's trip, it consists of motion in two different inertial frames and a very real lurch as he hops from the outbound ship to the returning Earth shuttle. Only Goslo remains in a single inertial frame, and so only he can correctly apply the simple time dilation formula of special relativity to Speedo's trip. Thus, Goslo finds that instead of aging 40 years ($20 \text{ ly}/0.500c$), Speedo actually ages only $(\sqrt{1 - v^2/c^2})(40 \text{ yr})$, or 34.6 yr. Clearly, Speedo spends 17.3 years going to planet X and 17.3 years returning in agreement with our earlier statement.

The result that Speedo ages 34.6 yr while Goslo ages 40 yr can be confirmed in a very direct experimental way from Speedo's frame if we use the special theory of relativity but take into account the fact that *Speedo's idealized trip takes place in two different inertial frames.* In yet another flight of fancy, suppose that Goslo celebrates his birthday each year in a flashy way, sending a powerful laser pulse to inform his twin that Goslo is another year older and wiser. Because Speedo is in an inertial frame on the outbound trip in which the Earth appears to be receding at $0.500c$, the flashes occur at a rate of one every

$$\frac{1}{\sqrt{1 - (v^2/c^2)}} \text{ yr} = \frac{1}{\sqrt{1 - [(0.500c)^2/c^2]}} \text{ yr} = 1.15 \text{ yr}$$

This occurs because moving clocks run slower. Also, because the Earth is receding, each successive flash must travel an additional distance of $(0.500c)(1.15 \text{ yr})$ between flashes. Consequently, Speedo observes flashes to arrive with a total time between flashes of $1.15 \text{ yr} + (0.500c)(1.15 \text{ yr})/c = 1.73$ yr. The total number of flashes seen by Speedo on his outbound voyage is therefore $(1 \text{ flash}/1.73 \text{ yr})(17.3 \text{ yr}) =$ 10 flashes. This means that Speedo views the Earth clocks to run more slowly than his own on the outbound trip because he observes 17.3 years to have passed for him while only 10 years have passed on Earth.

On the return voyage, because the Earth is racing toward Speedo with speed $0.500c$, successive flashes have less distance to travel, and the total time Speedo sees between the arrival of flashes is drastically shortened: $1.15 \text{ yr} - (0.500)(1.15 \text{ yr}) = 0.577 \text{ yr/flash}$. Thus, during the return trip, Speedo sees $(1 \text{ flash}/0.577 \text{ yr})(17.3 \text{ yr}) = 30$ flashes in total. In sum, during his 34.6 years of travel, Speedo receives $(10 + 30)$ flashes, indicating that his twin has aged 40 years. Notice that there has been no failure of special relativity for Speedo as long as we take his *two* inertial frames into account and assume negligible acceleration and deceleration times. On both the outbound and inbound trips Speedo correctly judges the Earth clocks to run slower than his own, but on the return trip his rapid movement toward the light flashes more than compensates for the slower rate of flashing.

Figure 1.16 "I love hearing that lonesome wail of the train whistle as the frequency of the wave changes due to the Doppler effect."

The Relativistic Doppler Shift

Another important consequence of time dilation is the shift in frequency found for light emitted by atoms in motion as opposed to light emitted by atoms at rest. A similar phenomenon, the mournful drop in pitch of the sound of a passing train's whistle, known as the Doppler effect, is quite familiar to most cowboys (Fig. 1.16). The Doppler shift for sound is usually studied in introductory physics courses and is especially interesting because motion of the source with respect to the medium of propagation can be clearly distinguished from motion of the observer. This means that in the case of sound we can distinguish the "absolute motion" of frames moving with respect to the air, which is the medium of propagation for sound.

Light waves must be analyzed differently from sound, because light waves require no medium of propagation and no method exists of distinguishing the motion of the light source from the motion of the observer. Thus, we expect to find a different formula for the Doppler shift of light waves, one that is only sensitive to the *relative* motion of source and observer and that holds for relative speeds of source and observer approaching c.

Consider a source of light waves at rest in frame S, emitting waves of frequency f and wavelength λ as measured in S. We wish to find the frequency f' and wavelength λ' of the light as measured by an observer fixed in frame S′, which is moving with speed v toward S, as shown in Figure 1.17a and b. In general, we expect f' to be greater than f if S′ approaches S because more wave crests are crossed per unit time, and we expect f' to be less than f if S′ recedes from S. In particular, consider the situation from the point of view of an observer fixed in S', as shown in Figure 1.18. This figure shows two successive wavefronts (color) emitted when the approaching source is at positions 1 and 2, respectively. If the time between the emission of these wavefronts as measured in S' is T', during this time front 1 will move a distance cT' from position 1. During this same time, the light source

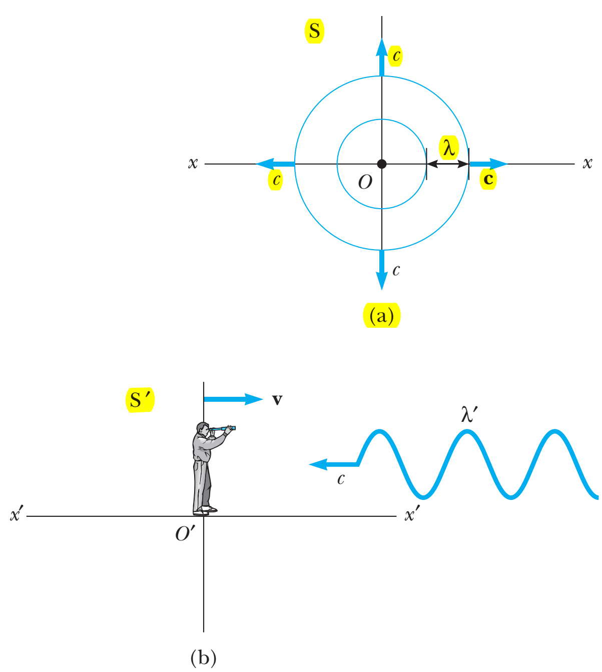

Figure 1.17 (a) A light source fixed in S emits wave crests separated in space by λ and moving outward at speed c as seen from S. (b) What wavelength λ' is measured by an observer at rest in S′? S′ is a frame approaching S at speed v such that the x- and x'-axes coincide.

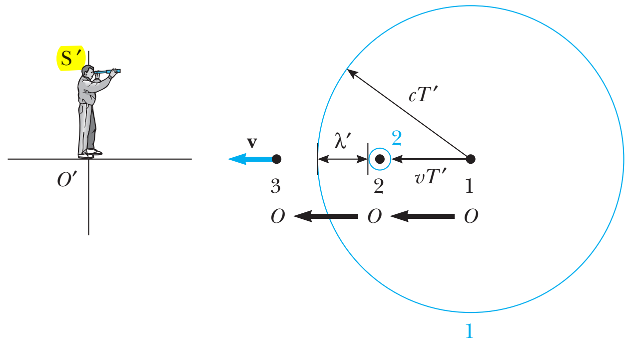

Figure 1.18 The view from S′. 1, 2, and 3 (in black) show three successive positions of O separated in time by T', the period of the light as measured from S′.

will advance a distance vT' to the left of position 1, and the distance between successive wavefronts will be measured in S′ to be

$$\lambda' = cT' - vT' \tag{1.11}$$

Because we wish to obtain a formula for f' (the frequency measured in S′) in terms of f (the frequency measured in S), we use the expression for λ' from Equation 1.11 in $f' = c/\lambda'$ to obtain

$$f' = \frac{c}{(c - v)T'} \tag{1.12}$$

To eliminate T' in favor of T, note that T is the proper time; that is, T is the time between two events (the emission of successive wavefronts) that occur at the same place in S, and consequently,

$$T' = \frac{T}{\sqrt{1 - (v^2/c^2)}}$$

Substituting for T' in Equation 1.12 and using $f = 1/T$ gives

$$f' = \frac{\sqrt{1 - (v^2/c^2)}}{1 - (v/c)} f \tag{1.13}$$

or

$$f' = \frac{\sqrt{1 + (v/c)}}{\sqrt{1 - (v/c)}} f \tag{1.14}$$

For clarity, this expression is often written

$$f_{\text{obs}} = \frac{\sqrt{1 + (v/c)}}{\sqrt{1 - (v/c)}} f_{\text{source}} \tag{1.15}$$

where f_{obs} is the frequency measured by an observer approaching a light source, and f_{source} is the frequency as measured in the source's rest frame.

Equation 1.15 is the relativistic Doppler shift formula, which, unlike the Doppler formula for sound, depends only on the relative speed v of the source and observer and holds for relative speeds as large as c. Equation 1.15 agrees with physical intuition in predicting f_{obs} to be greater than f_{source} for an approaching emitter and receiver. The expression for the case of a receding source is obtained by replacing v with $-v$ in Equation 1.15.

Although Christian Johann Doppler's name is most frequently associated with the effect in sound, he originally developed his ideas in an effort to understand the shift in frequency or wavelength of the light emitted by moving atoms and astronomical objects. The most spectacular and dramatic use of the Doppler effect has occurred in just this area in explaining the famous red shift of absorption lines (wavelengths) observed for most galaxies. (A galaxy is a cluster of millions of stars.) The term *redshift* refers to the shift of known absorption lines toward longer wavelengths, that is, toward the red end of the visible spectrum. For example, lines normally found in the extreme violet region for a galaxy at rest with respect to the Earth are shifted about 100 nm toward the red end of the spectrum for distant galaxies—indicating that these distant galaxies are rapidly *receding* from us. The American astronomer Edwin Hubble used this technique to confirm that most galaxies are moving away from us and that the Universe is *expanding*. (For more about the expanding Universe see Chapter 16, Cosmology, on our Web site.)

EXAMPLE 1.6 Determining the Speed of Recession of the Galaxy Hydra

The light emitted by a galaxy contains a continuous distribution of wavelengths because the galaxy is composed of millions of stars and other thermal emitters. However, some narrow gaps occur in the continuous spectrum where the radiation has been strongly absorbed by cooler gases in the galaxy. In particular, a cloud of ionized calcium atoms produces very strong absorption at 394 nm for a galaxy at rest with respect to the Earth. For the galaxy Hydra, which is 200 million ly away, this absorption is shifted to 475 nm. How fast is Hydra moving away from the Earth?

Solution For an approaching source and observer, $f_{obs} > f_{source}$ and $\lambda_{obs} < \lambda_{source}$ because $f_{obs}\lambda_{obs} = c = f_{source}\lambda_{source}$. In the case of Hydra, $\lambda_{obs} > \lambda_{source}$, so Hydra must be receding and we must use

$$f_{obs} = \frac{\sqrt{1 - (v/c)}}{\sqrt{1 + (v/c)}}\, f_{source}$$

Substituting $f_{obs} = c/\lambda_{obs}$ and $f_{source} = c/\lambda_{source}$ into this equation gives

$$\lambda_{obs} = \frac{\sqrt{1 + (v/c)}}{\sqrt{1 - (v/c)}}\, \lambda_{source}$$

Finally, solving for v/c, we find

$$\frac{v}{c} = \frac{\lambda_{obs}^2 - \lambda_{source}^2}{\lambda_{obs}^2 + \lambda_{source}^2}$$

or

$$\frac{v}{c} = \frac{(475\text{ nm})^2 - (394\text{ nm})^2}{(475\text{ nm})^2 + (394\text{ nm})^2} = 0.185$$

Therefore, Hydra is receding from us at $v = 0.185c = 5.54 \times 10^7$ m/s.

1.6 THE LORENTZ TRANSFORMATION

We have seen that the Galilean transformation is not valid when v approaches the speed of light. In this section, we shall derive the correct coordinate and velocity transformation equations that apply for all speeds in the range of $0 \leq v < c$. This transformation, known as the Lorentz transformation, was laboriously derived by Hendrik A. Lorentz (1853–1928, Dutch physicist) in 1890 as the transformation that made Maxwell's equations covariant. However, its real significance in a physical theory transcending electromagnetism was first recognized by Einstein.

The Lorentz coordinate transformation is a set of formulas that relates the space and time coordinates of two inertial observers moving with a relative speed v. We have already seen two consequences of the Lorentz transformation in the time dilation and length contraction formulas. The Lorentz velocity transformation is the set of formulas that relate the velocity components u_x, u_y, u_z of an object moving in frame S to the velocity components u'_x, u'_y, u'_z of the same object measured in frame S′, which is moving with a speed v relative to S. The Lorentz transformation formulas provide a formal, concise, and almost mechanical method of solution of relativity problems.

We start our derivation of the Lorentz transformation by noting that a reasonable guess (based on physical intuition) about the form of the coordinate equations can greatly reduce the algebraic complexity of the derivation. For simplicity, consider the standard frames, S and S′, with S′ moving at a speed v along the $+x$ direction (see Fig. 1.2). The origins of the two frames coincide at $t' = t = 0$. A reasonable guess about the dependence of x' on x and t is

$$x' = G(x - vt) \tag{1.16}$$

where G is a dimensionless factor that does not depend on x or t but is some function of v/c such that G is 1 in the limit as v/c approaches 0. The form of Equation 1.16 is suggested by the form of the Galilean transformation, $x' = x - vt$, which we know is correct in the limit as v/c approaches zero. The fact that Equation 1.16 is linear in x and t is also important because we require a single event in S (specified by x_1, t_1) to correspond to a single event in S′ (specified by x'_1, t'_1). Assuming that Equation 1.16 is correct, we can write the *inverse Lorentz coordinate transformation* for x in terms of x' and t' as

$$x = G(x' + vt') \tag{1.17}$$

This follows from Einstein's first postulate of relativity, which requires the laws of physics to have the same form in both S and S′ and where the sign of v has been changed to take into account the difference in direction of motion of the two frames. In fact, we should point out that this important technique for obtaining the inverse of a Lorentz transformation may be followed as a general rule:

> To obtain the inverse Lorentz transformation of any quantity, simply interchange primed and unprimed variables and reverse the sign of the frame velocity.

Returning to our derivation of the Lorentz transformations, our argument will be to take the differentials of x' and t' and form an expression that relates the measured velocity of an object in S′, $u'_x = dx'/dt'$, to the measured velocity of that object in S, $u_x = dx/dt$. We then determine G by requiring that u'_x must equal c in the case that u_x, the velocity of an object in frame S, is equal to c, in accord with Einstein's second postulate of relativity. Once G has been determined, this simple algebraic argument conveniently provides both the Lorentz coordinate and velocity transformations. Following this plan, we first find

$$t' = G\left\{t + (1/G^2 - 1)\frac{x}{v}\right\} \tag{1.18}$$

by substituting Equation 1.16 into 1.17 and solving for t'. Taking differentials of Equations 1.16 and 1.18 yields

$$dx' = G(dx - vdt) \tag{1.19}$$

$$dt' = G\left\{dt + (1/G^2 - 1)\frac{dx}{v}\right\} \tag{1.20}$$

Forming $u'_x = dx'/dt'$ leads, after some simplification, to

$$u'_x = \frac{dx'}{dt'} = \frac{u_x - v}{1 + (1/G^2 - 1)(u_x/v)} \tag{1.21}$$

where $u_x = dx/dt$.

Postulate 2 requires that the velocity of light be c for any observer, so in the case $u_x = c$, we must also have $u'_x = c$. Using this condition in Equation 1.21 gives

$$c = \frac{c - v}{1 + (1/G^2 - 1)(c/v)} \tag{1.22}$$

Equation 1.22 may be solved to give

$$G \equiv \gamma = \frac{1}{\sqrt{1 - (v^2/c^2)}}$$

The direct coordinate transformation is thus $x' = \gamma(x - vt)$, and the inverse transformation is $x = \gamma(x' + vt')$. To get the time transformation (t' as a function of t and x), substitute $G = \gamma$ into Equation 1.18 to obtain

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

In summary, the complete coordinate transformations between an event found to occur at (x, y, z, t) in S and (x', y', z', t') in S′ are

$$x' = \gamma(x - vt) \tag{1.23}$$

$$y' = y \tag{1.24}$$

$$z' = z \tag{1.25}$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \tag{1.26}$$

where

$$\gamma = \frac{1}{\sqrt{1 - (v^2/c^2)}}$$

Lorentz transformation for S → S′

If we wish to transform coordinates of an event in the S′ frame to coordinates in the S frame, we simply replace v by $-v$ and interchange the primed and unprimed coordinates in Equations 1.23 through 1.26. The resulting inverse transformation is given by

Inverse Lorentz transformation for S′ → S

$$\begin{aligned} x &= \gamma(x' - vt') \\ y &= y' \\ z &= z' \\ t &= \gamma\left(t' + \frac{vx'}{c^2}\right) \end{aligned} \tag{1.27}$$

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

In the Lorentz transformation, note that t depends on both t' and x'. Likewise, t' depends on both t and x. This is unlike the case of the Galilean transformation, in which $t = t'$. When $v \ll c$, the Lorentz transformation should reduce to the Galilean transformation. To check this, note that as $v \to 0$, $v/c < 1$ and $v^2/c^2 \ll 1$, so that Equations 1.23–1.26 reduce in this limit to the Galilean coordinate transformation equations, given by

$$x' = x - vt \qquad y' = y \qquad z' = z \qquad t' = t$$

EXAMPLE 1.7 Time Dilation Is Contained in the Lorentz Transformation

Show that the phenomenon of time dilation is contained in the Lorentz coordinate transformation. A light located at (x_0, y_0, z_0) is turned abruptly on at t_1 and off at t_2 in frame S. (a) For what time interval is the light measured to be on in frame S′? (See Figure 1.2 for a picture of the two standard frames.) (b) What is the distance between where the light is turned on and off as measured by S′?

Solution (a) The two events, the light turning on and the light turning off, are measured to occur in the two frames as follows:

	Event 1 (light on)	Event 2 (light off)
Frame S	x_0, t_1	x_0, t_2
Frame S′	$x_1' = \gamma(x_0 - vt_1)$	$x_2' = \gamma(x_0 - vt_2)$
	$t_1' = \gamma\left(t_1 - \frac{vx_0}{c^2}\right)$	$t_2' = \gamma\left(t_2 - \frac{vx_0}{c^2}\right)$

Note that the y and z coordinates are not affected because the motion of S′ is along x. As measured by S′, the light is on for a time interval

$$\begin{aligned} t_2' - t_1' &= \gamma\left(t_2 - \frac{vx_0}{c^2}\right) - \gamma\left(t_1 - \frac{vx_0}{c^2}\right) \\ &= \gamma(t_2 - t_1) \end{aligned}$$

Because $\gamma > 1$ and $(t_2 - t_1)$ is the proper time, it follows that $(t_2' - t_1') > (t_2 - t_1)$, and we have recovered our previous result for time dilation, Equation 1.8.

(b) Although event 1 and event 2 occur at the same place in S, they are measured to occur at a separation of $x_2' - x_1'$ in S′ where

$$\begin{aligned} x_2' - x_1' &= (\gamma x_0 - \gamma vt_2) - (\gamma x_0 - \gamma vt_1) \\ &= \gamma v(t_1 - t_2) \end{aligned}$$

This result is reasonable because it reduces to

$$v(t_1 - t_2) \quad \text{for } v/c \ll 1$$

Can you explain why $x_2' - x_1'$ is negative?

Exercise 5 Use the Lorentz transformation to derive the expression for length contraction. Note that the length of a moving object is determined by measuring the positions of both ends simultaneously.

Lorentz Velocity Transformation

The explicit form of the Lorentz velocity transformation follows immediately upon substitution of $G \equiv \gamma = 1/\sqrt{1 - (v^2/c^2)}$ into Equation 1.21:

Lorentz velocity transformation for S → S′

$$u'_x = \frac{u_x - v}{1 - (u_x v/c^2)} \tag{1.28}$$

where $u'_x = dx'/dt'$ is the instantaneous velocity in the x direction measured in S′ and $u_x = dx/dt$ is the velocity component u_x of the object as measured in S. Similarly, if the object has velocity components along y and z, the components in S′ are

$$u'_y = \frac{dy'}{dt'} = \frac{dy}{\gamma(dt - v\,dx/c^2)} = \frac{u_y}{\gamma[1 - (u_x v/c^2)]}$$
$$\text{and} \quad u'_z = \frac{u_z}{\gamma[1 - (u_x v/c^2)]} \tag{1.29}$$

When u_x and v are both much smaller than c (the nonrelativistic case), we see that the denominator of Equation 1.28 approaches unity, and so $u'_x \approx u_x - v$. This corresponds to the Galilean velocity transformation. In the other extreme, when $u_x = c$, Equation 1.28 becomes

$$u'_x = \frac{c - v}{1 - (cv/c^2)} = \frac{c[1 - (v/c)]}{1 - (v/c)} = c$$

From this result, we see that an object moving with a speed c relative to an observer in S also has a speed c relative to an observer in S′—*independent of the relative motion of S and S′*. Note that this conclusion is consistent with Einstein's second postulate, namely, that the speed of light must be c with respect to all inertial frames of reference. Furthermore, the speed of an object can never exceed c. That is, the speed of light is the "ultimate" speed. We return to this point later in Chapter 2 when we consider the energy of a particle.

To obtain u_x in terms of u'_x, replace v by $-v$ in Equation 1.28 and interchange u_x and u'_x following the rule stated earlier for obtaining the inverse transformation. This gives

Inverse Lorentz velocity transformation for S′ → S

$$u_x = \frac{u'_x + v}{1 + (u'_x v/c^2)} \tag{1.30}$$

EXAMPLE 1.8 Relative Velocity of Spaceships

Two spaceships A and B are moving in *opposite* directions, as in Figure 1.19. An observer on Earth measures the speed of A to be $0.750c$ and the speed of B to be $0.850c$. Find the velocity of B with respect to A.

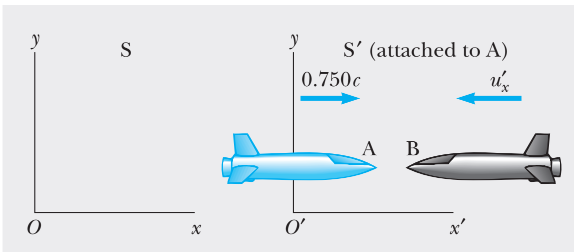

Figure 1.19 (Example 1.8) Two spaceships A and B move in *opposite* directions. The velocity of B relative to A is *less* than c and is obtained by using the relativistic velocity transformation.

Solution This problem can be solved by taking the S′ frame to be attached to spacecraft A, so that $v = 0.750c$ relative to an observer on Earth (the S frame). Spacecraft B can be considered as an object moving to the left with a velocity $u_x = -0.850c$ relative to the Earth observer. Hence, the velocity of B with respect to A can be obtained using Equation 1.28:

$$u'_x = \frac{u_x - v}{1 - \dfrac{u_x v}{c^2}} = \frac{-0.850c - 0.750c}{1 - \dfrac{(-0.850c)(0.750c)}{c^2}}$$

$$= -0.9771c$$

The negative sign for u'_x indicates that spaceship B is moving in the negative x direction as observed by A. Note that the result is less than c. That is, a body with speed less than c in one frame of reference must have a speed less than c in *any other* frame. If the incorrect Galilean velocity transformation were used in this example, we would find that $u'_x = u_x - v = -0.850c - 0.750c = -1.600c$, which is greater than the universal limiting speed c.

EXAMPLE 1.9 The Speeding Motorcycle

Imagine a motorcycle rider moving with a speed of $0.800c$ past a stationary observer, as shown in Figure 1.20. If the rider tosses a ball in the forward direction with a speed of $0.700c$ with respect to himself, what is the speed of the ball as seen by the stationary observer?

Solution In this situation, the velocity of the motorcycle with respect to the stationary observer is $v = 0.800c$. The velocity of the ball in the frame of reference of the motorcyclist is $u'_x = 0.700c$. Therefore, the velocity, u_x, of

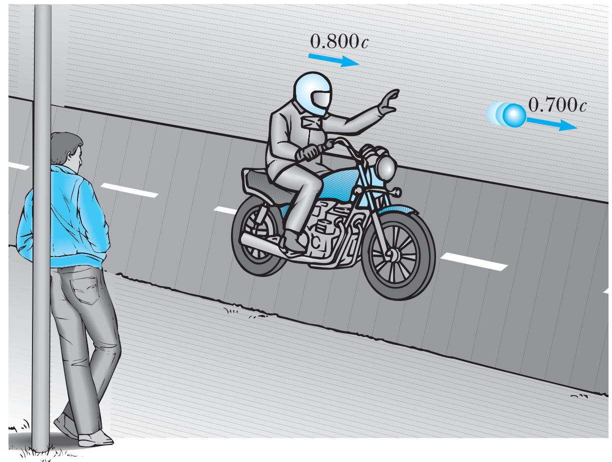

Figure 1.20 (Example 1.9) A motorcyclist moves past a stationary observer with a speed of $0.800c$ and throws a ball in the direction of motion with a speed of $0.700c$ relative to himself.

the ball relative to the stationary observer is

$$u_x = \frac{u'_x + v}{1 + (u'_x v/c^2)}$$

$$= \frac{0.700c + 0.800c}{1 + [(0.700c)(0.800c)/c^2]} = 0.9615c$$

Exercise 6 Suppose that the motorcyclist moving with a speed $0.800c$ turns on a beam of light that moves away from him with a speed of c in the same direction as the moving motorcycle. What would the stationary observer measure for the speed of the beam of light?

Answer c.

EXAMPLE 1.10 Relativistic Leaders of the Pack!

Imagine two motorcycle gang leaders racing at relativistic speeds along perpendicular paths from the local pool hall, as shown in Figure 1.21. How fast does pack leader Beta recede over Alpha's right shoulder as seen by Alpha?

Solution Figure 1.21 shows the situation as seen by a stationary police officer located in frame S, who observes the following:

Pack Leader Alpha	$u_x = 0.75c$	$u_y = 0$
Pack Leader Beta	$u_x = 0$	$u_y = -0.90c$

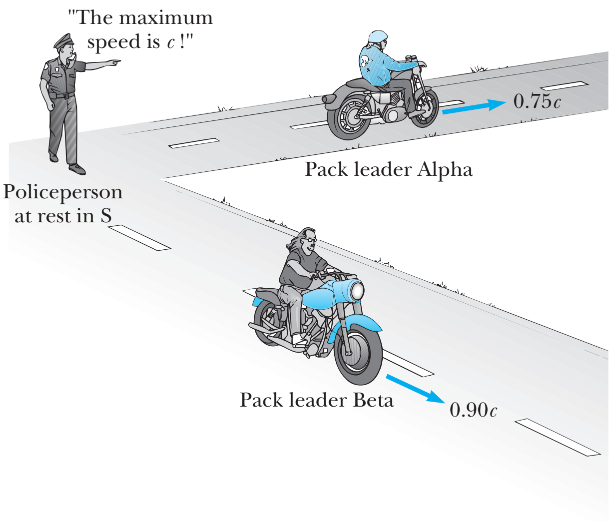

Figure 1.21 (Example 1.10) Two motorcycle pack leaders, Alpha and Beta, blaze past a stationary police officer. They are leading their respective gangs from the pool hall along perpendicular roads.

To get Beta's speed of recession as seen by Alpha, we take S′ to move along with Alpha, as shown in Figure 1.22, and we calculate u'_x and u'_y for Beta using Equations 1.28 and 1.29:

$$u'_x = \frac{u_x - v}{1 - (u_x v/c^2)} = \frac{0 - 0.75c}{1 - [(0)(0.75c)/c^2]} = -0.75c$$

$$u'_y = \frac{u_y}{\gamma[1 - (u_x v/c^2)]}$$

$$= \frac{\sqrt{1 - [(0.75c)^2/c^2]}(-0.90c)}{1 - [(0)(0.75c)/c^2]} = -0.60c$$

The speed of recession of Beta away from Alpha as observed by Alpha is then found to be less than c as required by relativity.

$$u' = \sqrt{(u'_x)^2 + (u'_y)^2} = \sqrt{(-0.75c)^2 + (-0.60c)^2} = 0.96c$$

Exercise 7 Calculate the classical speed of recession of Beta from Alpha using the incorrect Galilean transformation.

Answer 1.2c

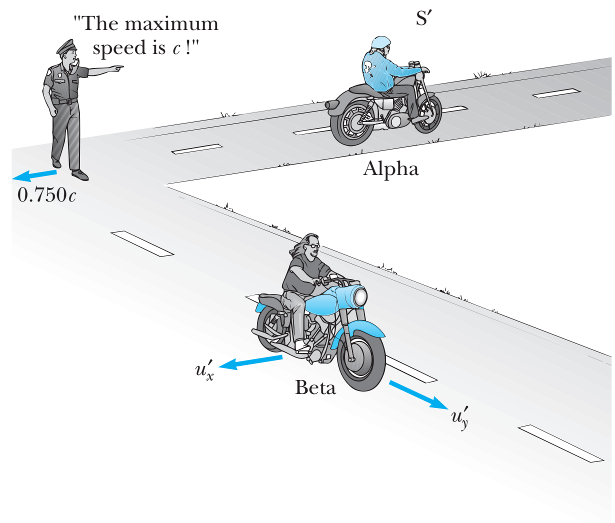

Figure 1.22 (Example 1.10) Pack leader Alpha's view of things.

1.7 SPACETIME AND CAUSALITY

The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a union of the two will preserve an independent reality. (Hermann Minkowski, 1908, in an address to the Assembly of German Natural Scientists and Physicians)

We have seen in relativity that space and time coordinates cannot be treated separately. This is apparent from both the combination of space and time coordinates required in the Lorentz coordinate transformation and in the variation of length and time intervals with inertial frame as shown in the time dilation and length contraction formulas. A convenient way to express the entanglement of space and time is with the concept of four-dimensional *spacetime*