

Chapter 16 (Waves I)

6. (a) The amplitude is $y_m = 6.0$ cm.

(b) We find λ from $2\pi/\lambda = 0.020\pi$. $\lambda = 1.0 \times 10^2$ cm.

(c) Solving $2\pi f = \omega = 4.0\pi$, we obtain $f = 2.0$ Hz.

(d) The wave speed is $v = \lambda f = (100 \text{ cm})(2.0 \text{ Hz}) = 2.0 \times 10^2$ cm/s.

(e) The wave propagates in the $-x$ direction, since the argument of the trig function is $kx + \omega t$ instead of $kx - \omega t$ (as in Eq. 16-2).

(f) The maximum transverse speed (found from the time derivative of y) is

$$u_{\max} = 2\pi f y_m = (4.0\pi \text{ s}^{-1})(6.0 \text{ cm}) = 75 \text{ cm/s}.$$

(g) $y(3.5 \text{ cm}, 0.26 \text{ s}) = (6.0 \text{ cm}) \sin[0.020\pi(3.5) + 4.0\pi(0.26)] = -2.0 \text{ cm}$.

19. (a) We read the amplitude from the graph. It is about 5.0 cm.

(b) We read the wavelength from the graph. The curve crosses $y = 0$ at about $x = 15$ cm and again with the same slope at about $x = 55$ cm, so

$$\lambda = (55 \text{ cm} - 15 \text{ cm}) = 40 \text{ cm} = 0.40 \text{ m}.$$

(c) The wave speed is $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Thus,

$$v = \sqrt{\frac{3.6 \text{ N}}{25 \times 10^{-3} \text{ kg/m}}} = 12 \text{ m/s}.$$

(d) The frequency is $f = v/\lambda = (12 \text{ m/s})/(0.40 \text{ m}) = 30$ Hz and the period is

$$T = 1/f = 1/(30 \text{ Hz}) = 0.033 \text{ s}.$$

(e) The maximum string speed is

$$u_m = \omega y_m = 2\pi f y_m = 2\pi(30 \text{ Hz})(5.0 \text{ cm}) = 940 \text{ cm/s} = 9.4 \text{ m/s}.$$

(f) The angular wave number is $k = 2\pi/\lambda = 2\pi/(0.40 \text{ m}) = 16 \text{ m}^{-1}$.

(g) The angular frequency is $\omega = 2\pi f = 2\pi(30 \text{ Hz}) = 1.9 \times 10^2 \text{ rad/s}$

(h) According to the graph, the displacement at $x = 0$ and $t = 0$ is 4.0×10^{-2} m. The formula for the displacement gives $y(0, 0) = y_m \sin \phi$. We wish to select ϕ so that $5.0 \times$

$10^{-2} \sin \phi = 4.0 \times 10^{-2}$. The solution is either 0.93 rad or 2.21 rad. In the first case the function has a positive slope at $x = 0$ and matches the graph. In the second case it has negative slope and does not match the graph. We select $\phi = 0.93$ rad.

(i) The string displacement has the form $y(x, t) = y_m \sin(kx + \omega t + \phi)$. A plus sign appears in the argument of the trigonometric function because the wave is moving in the negative x direction. Using the results obtained above, the expression for the displacement is

$$y(x, t) = (5.0 \times 10^{-2} \text{ m}) \sin[(16 \text{ m}^{-1})x + (190 \text{ s}^{-1})t + 0.93].$$

25. We note from the graph (and from the fact that we are dealing with a cosine-squared, see Eq. 16-30) that the wave frequency is $f = \frac{1}{2 \text{ ms}} = 500$ Hz, and that the

wavelength $\lambda = 0.20$ m. We also note from the graph that the maximum value of dK/dt is 10 W. Setting this equal to the maximum value of Eq. 16-29 (where we just set that cosine term equal to 1) we find

$$\frac{1}{2} \mu v \omega^2 y_m^2 = 10$$

with SI units understood. Substituting in $\mu = 0.002$ kg/m, $\omega = 2\pi f$ and $v = f\lambda$, we solve for the wave amplitude:

$$y_m = \sqrt{\frac{10}{2\pi^2 \mu \lambda f^3}} = 0.0032 \text{ m}.$$

31. (a) The amplitude of the second wave is $y_m = 9.00$ mm, as stated in the problem.

(b) The figure indicates that $\lambda = 40$ cm = 0.40 m, which implies that the angular wave number is $k = 2\pi/0.40 = 16$ rad/m.

(c) The figure (along with information in the problem) indicates that the speed of each wave is $v = dx/t = (56.0 \text{ cm})/(8.0 \text{ ms}) = 70$ m/s. This, in turn, implies that the angular frequency is

$$\omega = kv = 1100 \text{ rad/s} = 1.1 \times 10^3 \text{ rad/s}.$$

(d) We observe that Figure 16-38 depicts two traveling waves (both going in the $-x$ direction) of equal amplitude y_m . The amplitude of their resultant wave, as shown in the figure, is $y'_m = 4.00$ mm. Eq. 16-52 applies:

$$y'_m = 2 y_m \cos(1/2 \phi_2) \Rightarrow \phi_2 = 2 \cos^{-1}(2.00/9.00) = 2.69 \text{ rad}.$$

(e) In making the plus-or-minus sign choice in $y = y_m \sin(kx \pm \omega t + \phi)$, we recall the discussion in section 16-5, where it is shown that sinusoidal waves traveling in the $-x$ direction are of the form $y = y_m \sin(kx + \omega t + \phi)$. Here, ϕ should be thought of as the phase *difference* between the two waves (that is, $\phi_1 = 0$ for wave 1 and $\phi_2 = 2.69$ rad for wave 2).

In summary, the waves have the forms (with SI units understood):

$$y_1 = (0.00900)\sin(16x + 1100t) \quad \text{and} \quad y_2 = (0.00900)\sin(16x + 1100t + 2.7).$$

49. (a) The waves have the same amplitude, the same angular frequency, and the same angular wave number, but they travel in opposite directions. We take them to be $y_1 = y_m \sin(kx - \omega t)$ and $y_2 = y_m \sin(kx + \omega t)$. The amplitude y_m is half the maximum displacement of the standing wave, or 5.0×10^{-3} m.

(b) Since the standing wave has three loops, the string is three half-wavelengths long: $L = 3\lambda/2$, or $\lambda = 2L/3$. With $L = 3.0$ m, $\lambda = 2.0$ m. The angular wave number is $k = 2\pi/\lambda = 2\pi/(2.0 \text{ m}) = 3.1 \text{ m}^{-1}$.

(c) If v is the wave speed, then the frequency is

$$f = \frac{v}{\lambda} = \frac{3v}{2L} = \frac{3(100 \text{ m/s})}{2(3.0 \text{ m})} = 50 \text{ Hz}.$$

The angular frequency is the same as that of the standing wave, or $\omega = 2\pi f = 2\pi(50 \text{ Hz}) = 314 \text{ rad/s}$.

(d) The two waves are

$$y_1 = (5.0 \times 10^{-3} \text{ m}) \sin \left[(3.14 \text{ m}^{-1})x - (314 \text{ s}^{-1})t \right]$$

and

$$y_2 = (5.0 \times 10^{-3} \text{ m}) \sin \left[(3.14 \text{ m}^{-1})x + (314 \text{ s}^{-1})t \right].$$

Thus, if one of the waves has the form $y(x,t) = y_m \sin(kx + \omega t)$, then the other wave must have the form $y'(x,t) = y_m \sin(kx - \omega t)$. The sign in front of ω for $y'(x,t)$ is minus.