

10. We use Eq. 8-17, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

- (a) In the solution to exercise 2 (to which this problem refers), we found  $U_i = mgy_i = 196$  J and  $U_f = mgy_f = 29$  J (assuming the reference position is at the ground). Since  $K_i = 0$  in this case, we have

$$\begin{aligned}K_i + U_i &= K_f + U_f \\0 + 196 &= K_f + 29\end{aligned}$$

which gives  $K_f = 167$  J and thus leads to

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(167)}{2.00}} = 12.9 \text{ m/s} .$$

- (b) If we proceed algebraically through the calculation in part (a), we find  $K_f = -\Delta U = mgh$  where  $h = y_i - y_f$  and is positive-valued. Thus,

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{2gh}$$

as we might also have derived from the equations of Table 2-1 (particularly Eq. 2-16). The fact that the answer is independent of mass means that the answer to part (b) is identical to that of part (a).

- (c) If  $K_i \neq 0$ , then we find  $K_f = mgh + K_i$  (where  $K_i$  is necessarily positive-valued). This represents a larger value for  $K_f$  than in the previous parts, and thus leads to a larger value for  $v$ .

22. Since time does not directly enter into the energy formulations, we return to Chapter 4 (or Table 2-1 in Chapter 2) to find the change of height during this  $t = 6$  s flight.

$$\Delta y = v_{0y}t - \frac{1}{2}gt^2$$

This leads to  $\Delta y = -32$  m. Therefore  $\Delta U = mg\Delta y = -318 \approx -320$  J.

33. The work required is the change in the gravitational potential energy as a result of the chain being pulled onto the table. Dividing the hanging chain into a large number of infinitesimal segments, each of length  $dy$ , we note that the mass of a segment is  $(m/L) dy$  and the change in potential energy of a segment when it is a distance  $|y|$  below the table top is  $dU = (m/L)g|y| dy = -(m/L)gy dy$  since  $y$  is negative-valued (we have  $+y$  upward and the origin is at the tabletop). The total potential energy change is

$$\Delta U = -\frac{mg}{L} \int_{-L/4}^0 y dy = \frac{1}{2} \frac{mg}{L} (L/4)^2 = mgL/32 .$$

The work required to pull the chain onto the table is therefore  $W = \Delta U = mgL/32$ .

44. We use SI units so  $m = 0.030$  kg and  $d = 0.12$  m.

- (a) Since there is no change in height (and we assume no changes in elastic potential energy), then  $\Delta U = 0$  and we have

$$\Delta E_{\text{mech}} = \Delta K = -\frac{1}{2}mv_0^2 = -3.8 \times 10^3 \text{ J}$$

where  $v_0 = 500$  m/s and the final speed is zero.

- (b) By Eq. 8-31 (with  $W = 0$ ) we have  $\Delta E_{\text{th}} = 3.8 \times 10^3$  J, which implies

$$f = \frac{\Delta E_{\text{th}}}{d} = 3.1 \times 10^4 \text{ N}$$

using Eq. 8-29 with  $f_k$  replaced by  $f$  (effectively generalizing that equation to include a greater variety of dissipative forces than just those obeying Eq. 6-2).

69. Let the amount of stretch of the spring be  $x$ . For the object to be in equilibrium

$$kx - mg = 0 \implies x = mg/k .$$

Thus the gain in elastic potential energy for the spring is

$$\Delta U_e = \frac{1}{2}kx^2 = \frac{1}{2}k \left( \frac{mg}{k} \right)^2 = \frac{m^2 g^2}{2k}$$

while the loss in the gravitational potential energy of the system is

$$-\Delta U_g = mgx = mg \left( \frac{mg}{k} \right) = \frac{m^2 g^2}{k}$$

which we see (by comparing with the previous expression) is equal to  $2\Delta U_e$ . The reason why  $|\Delta U_g| \neq \Delta U_e$  is that, since the object is slowly lowered, an upward external force (e.g., due to the hand) must have been exerted on the object during the lowering process, preventing it from accelerating downward. This force does *negative* work on the object, reducing the total mechanical energy of the system.