12. (a) Making sure our calculator is in radians mode, we find

$$x = 6.0 \cos\left(3\pi(2.0) + \frac{\pi}{3}\right) = 3.0 \text{ m} .$$

(b) Differentiating with respect to time and evaluating at $t = 2.0$ s, we find

$$v = \frac{dx}{dt} = -3\pi(6.0)\sin\left(3\pi(2.0) + \frac{\pi}{3}\right) = -49 \text{ m/s} .$$

(c) Differentiating again, we obtain

$$a = \frac{dv}{dt} = -(3\pi)^2(6.0)\cos\left(3\pi(2.0) + \frac{\pi}{3}\right) = -2.7 \times 10^2 \text{ m/s}^2 .$$

(d) In the second paragraph after Eq. 16-3, the textbook defines the phase of the motion. In this case (with $t = 2.0$ s) the phase is $3\pi(2.0) + \frac{\pi}{3} \approx 20$ rad.

(e) Comparing with Eq. 16-3, we see that $\omega = 3\pi$ rad/s. Therefore, $f = \omega/2\pi = 1.5$ Hz.

(f) The period is the reciprocal of the frequency: $T = 1/f \approx 0.67$ s.

18. Both parts of this problem deal with the critical case when the maximum acceleration becomes equal to that of free fall. The textbook notes (in the discussion immediately after Eq. 16-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency; this is the expression we set equal to $g = 9.8\,\mathrm{m/s^2}$.

(a) Using Eq. 16-5 and $T = 1.0$ s, we have

$$\left(\frac{2\pi}{T}\right)^2 x_m = g \implies x_m = \frac{gT^2}{4\pi^2} = 0.25 \text{ m} .$$

(b) Since $\omega = 2\pi f$, and $x_m = 0.050$ m is given, we find

$$\left(2\pi f)^2\right) x_m = g \implies f = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = 2.2 \text{ Hz} .$$

54. Since the centripetal acceleration is horizontal and Earth's gravitational $\vec{g}$ is downward, we can define the magnitude of an "effective" gravitational acceleration using the Pythagorean theorem:

$$g_{\text{eff}} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2} \ .$$

Then, since frequency is the reciprocal of the period, Eq. 16-28 leads to

$$f = \frac{1}{2\pi}\sqrt{\frac{g_{\text{eff}}}{L}} = \frac{1}{2\pi}\sqrt{\frac{\sqrt{g^2 + v^4/R^2}}{L}} \ .$$