6. The magnitude F of the force required to pull the lid off is $F = (p_o - p_i)A$, where p_o is the pressure outside the box, p_i is the pressure inside, and A is the area of the lid. Recalling that $1\,\mathrm{N/m^2} = 1\,\mathrm{Pa}$, we obtain

$$p_i = p_o - \frac{F}{A} = 1.0 \times 10^5\,\mathrm{Pa} - \frac{480\,\mathrm{N}}{77 \times 10^{-4}\,\mathrm{m^2}} = 3.8 \times 10^4\ \mathrm{Pa}\ .$$

19. (a) At depth y the gauge pressure of the water is $p = \rho g y$, where ρ is the density of the water. We consider a horizontal strip of width W at depth y, with (vertical) thickness dy, across the dam. Its area is $dA = W\,dy$ and the force it exerts on the dam is $dF = p\,dA = \rho g y W\,dy$. The total force of the water on the dam is

$$F = \int_0^D \rho g y W\,dy = \frac{1}{2}\rho g W D^2 \ .$$

(b) Again we consider the strip of water at depth y. Its moment arm for the torque it exerts about O is $D - y$ so the torque it exerts is $d\tau = dF(D-y) = \rho g y W(D-y)dy$ and the total torque of the water is

$$\tau = \int_0^D \rho g y W(D-y)\,dy = \rho g W\left(\frac{1}{2}D^3 - \frac{1}{3}D^3\right) = \frac{1}{6}\rho g W D^3 \ .$$

(c) We write $\tau = rF$, where r is the effective moment arm. Then,

$$r = \frac{\tau}{F} = \frac{\frac{1}{6}\rho g W D^3}{\frac{1}{2}\rho g W D^2} = \frac{D}{3} \ .$$

32. (a) Since the lead is not displacing any water (of density ρ_w), the lead's volume is not contributing to the buoyant force F_b. If the immersed volume of wood is V_i, then

$$F_b = \rho_w V_i g = 0.90 \rho_w V_{\text{wood}} g = 0.90 \rho_w g \left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}} \right) ,$$

which, when floating, equals the weights of the wood and lead:

$$F_b = 0.90 \rho_w g \left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}} \right) = (m_{\text{wood}} + m_{\text{lead}}) g \; .$$

Thus,

$$\begin{aligned} m_{\text{lead}} &= 0.90 \rho_w \left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}} \right) - m_{\text{wood}} \\ &= \frac{(0.90)(1000\,\text{kg/m}^3)(3.67\,\text{kg})}{600\,\text{kg/m}^3} - 3.67\,\text{kg} = 1.84\,\text{kg} \; \approx 1.8 \text{ kg} \; . \end{aligned}$$

(b) In this case, the volume $V_{\text{lead}} = m_{\text{lead}}/\rho_{\text{lead}}$ also contributes to F_b. Consequently,

$$F_b = 0.90 \rho_w g \left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}} \right) + \left(\frac{\rho_w}{\rho_{\text{lead}}} \right) m_{\text{lead}} g = (m_{\text{wood}} + m_{\text{lead}}) g \, ,$$

which leads to

$$\begin{aligned} m_{\text{lead}} &= \frac{0.90 (\rho_w/\rho_{\text{wood}}) m_{\text{wood}} - m_{\text{wood}}}{1 - \rho_w/\rho_{\text{lead}}} \\ &= \frac{1.84\,\text{kg}}{1 - \left(1.00 \times 10^3 \text{ kg/m}^3 / 1.13 \times 10^4 \text{ kg/m}^3 \right)} = 2.0 \text{ kg} \; . \end{aligned}$$

41. Suppose that a mass Δm of water is pumped in time Δt. The pump increases the potential energy of the water by Δmgh, where h is the vertical distance through which it is lifted, and increases its kinetic energy by $\frac{1}{2}\Delta mv^2$, where v is its final speed. The work it does is $\Delta W = \Delta mgh + \frac{1}{2}\Delta mv^2$ and its power is

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta m}{\Delta t}\left(gh + \frac{1}{2}v^2\right) .$$

Now the rate of mass flow is $\Delta m/\Delta t = \rho_w Av$, where ρ_w is the density of water and A is the area of the hose. The area of the hose is $A = \pi r^2 = \pi(0.010\,\text{m})^2 = 3.14 \times 10^{-4}\,\text{m}^2$ and $\rho_w Av = (1000\,\text{kg/m}^3)(3.14 \times 10^{-4}\,\text{m}^2)(5.0\,\text{m/s}) = 1.57\,\text{kg/s}$. Thus,

$$\begin{aligned} P &= \rho Av\left(gh + \frac{1}{2}v^2\right) \\ &= (1.57\,\text{kg/s})\left((9.8\,\text{m/s}^2)(3.0\,\text{m}) + \frac{(5.0\,\text{m/s})^2}{2}\right) = 66\ \text{W} . \end{aligned}$$

54. (a) Since Sample Problem 15-9 deals with a similar situation, we use the final equation (labeled "Answer") from it:

$$v = \sqrt{2gh} \quad \Longrightarrow \quad v = v_o \text{ for the projectile motion.}$$

The stream of water emerges horizontally ($\theta_o = 0°$ in the notation of Chapter 4), and setting $y - y_o = -(H - h)$ in Eq. 4-22, we obtain the "time-of-flight"

$$t = \sqrt{\frac{-2(H-h)}{-g}} = \sqrt{\frac{2}{g}(H-h)} \ .$$

Using this in Eq. 4-21, where $x_o = 0$ by choice of coordinate origin, we find

$$x = v_o t = \sqrt{2gh}\sqrt{\frac{2}{g}(H-h)} = 2\sqrt{h(H-h)} \ .$$

(b) The result of part (a) (which, when squared, reads $x^2 = 4h(H - h)$) is a quadratic equation for h once x and H are specified. Two solutions for h are therefore mathematically possible, but are they both physically possible? For instance, are both solutions positive and less than H? We employ the quadratic formula:

$$h^2 - Hh + \frac{x^2}{4} = 0 \quad \Longrightarrow \quad h = \frac{H \pm \sqrt{H^2 - x^2}}{2}$$

which permits us to see that both roots are physically possible, so long as $x < H$. Labeling the larger root h_1 (where the plus sign is chosen) and the smaller root as h_2 (where the minus sign is chosen), then we note that their sum is simply

$$h_1 + h_2 = \frac{H + \sqrt{H^2 - x^2}}{2} + \frac{H - \sqrt{H^2 - x^2}}{2} = H \ .$$

Thus, one root is related to the other (generically labeled h' and h) by $h' = H - h$.

(c) We wish to maximize the function $f = x^2 = 4h(H - h)$. We differentiate with respect to h and set equal to zero to obtain

$$\frac{df}{dh} = 4H - 8h = 0 \quad \Longrightarrow \quad h = \frac{H}{2}$$

as the depth from which an emerging stream of water will travel the maximum horizontal distance.