5. The gravitational force between the two parts is

$$F = \frac{Gm(M - m)}{r^2} = \frac{G}{r^2}\left(mM - m^2\right)$$

which we differentiate with respect to m and set equal to zero:

$$\frac{dF}{dm} = 0 = \frac{G}{r^2}(M - 2m) \implies M = 2m$$

which leads to the result $m/M = 1/2$.

8. Using $F = GmM/r^2$, we find that the topmost mass pulls upward on the one at the origin with 1.9×10^{-8} N, and the rightmost mass pulls rightward on the one at the origin with 1.0×10^{-8} N. Thus, the (x, y) components of the net force, which can be converted to polar components (here we use magnitude-angle notation), are

$$\vec{F}_{\text{net}} = \left(1.0 \times 10^{-8}, 1.9 \times 10^{-8}\right) \implies \left(2.1 \times 10^{-8} \angle\ 61^\circ\right) .$$

The magnitude of the force is 2.1×10^{-8} N.

21. From Eq. 14-13, we see the extreme case is when "g" becomes zero, and plugging in Eq. 14-14 leads to

$$0 = \frac{GM}{R^2} - R\omega^2 \implies M = \frac{R^3\omega^2}{G} .$$

Thus, with $R = 20000$ m and $\omega = 2\pi$ rad/s, we find $M = 4.7 \times 10^{24}$ kg.

33. (a) We use the principle of conservation of energy. Initially the rocket is at Earth's surface and the potential energy is $U_i = -GMm/R_e = -mgR_e$, where M is the mass of Earth, m is the mass of the rocket, and R_e is the radius of Earth. The relationship $g = GM/R_e^2$ was used. The initial kinetic energy is $\frac{1}{2}mv^2 = 2mgR_e$, where the substitution $v = 2\sqrt{gR_e}$ was made. If the rocket can escape then conservation of energy must lead to a positive kinetic energy no matter how far from Earth it gets. We take the final potential energy to be zero and let K_f be the final kinetic energy. Then, $U_i + K_i = U_f + K_f$ leads to $K_f = U_i + K_i = -mgR_e + 2mgR_e = mgR_e$. The result is positive and the rocket has enough kinetic energy to escape the gravitational pull of Earth.

(b) We write $\frac{1}{2}mv_f^2$ for the final kinetic energy. Then, $\frac{1}{2}mv_f^2 = mgR_e$ and $v_f = \sqrt{2gR_e}$.

45. (a) If r is the radius of the orbit then the magnitude of the gravitational force acting on the satellite is given by GMm/r^2, where M is the mass of Earth and m is the mass of the satellite. The magnitude of the acceleration of the satellite is given by v^2/r, where v is its speed. Newton's second law yields $GMm/r^2 = mv^2/r$. Since the radius of Earth is 6.37×10^6 m the orbit radius is $r = 6.37 \times 10^6\text{ m} + 160 \times 10^3\text{ m} = 6.53 \times 10^6\text{ m}$. The solution for v is

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11}\text{ m}^3/\text{s}^2\cdot\text{kg})(5.98 \times 10^{24}\text{ kg})}{6.53 \times 10^6\text{ m}}} = 7.82 \times 10^3\text{ m/s} .$$

(b) Since the circumference of the circular orbit is $2\pi r$, the period is

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.53 \times 10^6\text{ m})}{7.82 \times 10^3\text{ m/s}} = 5.25 \times 10^3\text{ s} .$$

This is equivalent to 87.4 min.

61. The energy required to raise a satellite of mass m to an altitude h (at rest) is given by

$$E_1 = \Delta U = GM_E m\left(\frac{1}{R_E} - \frac{1}{R_E + h}\right) ,$$

and the energy required to put it in circular orbit once it is there is

$$E_2 = \frac{1}{2}mv_{\text{orb}}^2 = \frac{GM_E m}{2(R_E + h)} .$$

Consequently, the energy difference is

$$\Delta E = E_1 - E_2 = GM_E m\left[\frac{1}{R_E} - \frac{3}{2(R_E + h)}\right] .$$

(a) Since

$$\frac{1}{R_E} - \frac{3}{2(R_E + h)} = \frac{1}{6370\,\text{km}} - \frac{3}{2(6370\,\text{km} + 1500\,\text{km})} < 0$$

the answer is no ($E_1 < E_2$).

(b) Since

$$\frac{1}{R_E} - \frac{3}{2(R_E + h)} = \frac{1}{6370\,\text{km}} - \frac{3}{2(6370\,\text{km} + 3185\,\text{km})} = 0$$

we have $E_1 = E_2$.

(c) Since

$$\frac{1}{R_E} - \frac{3}{2(R_E + h)} = \frac{1}{6370\,\text{km}} - \frac{3}{2(6370\,\text{km} + 4500\,\text{km})} > 0$$

the answer is yes ($E_1 > E_2$).