15. (a) The derivation of the acceleration is found in §12-4; Eq. 12-13 gives

$$a_{\text{com}} = -\frac{g}{1 + I_{\text{com}}/MR_0^2}$$

where the positive direction is upward. We use $I_{\text{com}} = 950\,\text{g}\cdot\text{cm}^2$, $M = 120\,\text{g}$, $R_0 = 0.32\,\text{cm}$ and $g = 980\,\text{cm/s}^2$ and obtain

$$|a_{\text{com}}| = \frac{980}{1 + (950)/(120)(0.32)^2} = 12.5\ \text{cm/s}^2\ .$$

(b) Taking the coordinate origin at the initial position, Eq. 2-15 leads to $y_{\text{com}} = \frac{1}{2}a_{\text{com}}t^2$. Thus, we set $y_{\text{com}} = -120\,\text{cm}$, and find

$$t = \sqrt{\frac{2y_{\text{com}}}{a_{\text{com}}}} = \sqrt{\frac{2(-120\,\text{cm})}{-12.5\,\text{cm/s}^2}} = 4.38\ \text{s}\ .$$

(c) As it reaches the end of the string, its center of mass velocity is given by Eq. 2-11: $v_{\text{com}} = a_{\text{com}}t = (-12.5\,\text{cm/s}^2)(4.38\,\text{s}) = -54.8\,\text{cm/s}$, so its linear speed then is approximately 55 cm/s.

(d) The translational kinetic energy is $\frac{1}{2}mv_{\text{com}}^2 = \frac{1}{2}(0.120\,\text{kg})(0.548\,\text{m/s})^2 = 1.8 \times 10^{-2}\,\text{J}$.

(e) The angular velocity is given by $\omega = -v_{\text{com}}/R_0$ and the rotational kinetic energy is

$$\frac{1}{2}I_{\text{com}}\omega^2 = \frac{1}{2}I_{\text{com}}\frac{v_{\text{com}}^2}{R_0^2} = \frac{1}{2}\frac{(9.50 \times 10^{-5}\,\text{kg}\cdot\text{m}^2)(0.548\,\text{m/s})^2}{(3.2 \times 10^{-3}\,\text{m})^2}$$

which yields $K_{\text{rot}} = 1.4$ J.

(f) The angular speed is $\omega = |v_{\text{com}}|/R_0 = (0.548\,\text{m/s})/(3.2 \times 10^{-3}\,\text{m}) = 1.7 \times 10^2\,\text{rad/s} = 27\,\text{rev/s}$.

23. We could proceed formally by setting up an xyz coordinate system and using Eq. 3-30 for the vector cross product, or we can approach this less formally in the style of Sample Problem 12-4 (which is our choice). For the 3.1 kg particle, Eq. 12-21 yields

$$\ell_1 = r_{\perp\,1}\, m v_1 = (2.8)(3.1)(3.6) = 31.2 \text{ kg}\cdot\text{m}^2/\text{s} .$$

Using the right-hand rule for vector products, we find this $(\vec{r}_1 \times \vec{p}_1)$ is out of the page, perpendicular to the plane of Fig. 12-35. And for the 6.5 kg particle, we find

$$\ell_2 = r_{\perp\,2}\, m v_2 = (1.5)(6.5)(2.2) = 21.4 \text{ kg}\cdot\text{m}^2/\text{s} .$$

And we use the right-hand rule again, finding that this $(\vec{r}_2 \times \vec{p}_2)$ is into the page. Consequently, the two angular momentum vectors are in opposite directions, so their vector sum is the *difference* of their magnitudes:

$$L = \ell_1 - \ell_2 = 9.8 \text{ kg}\cdot\text{m}^2/\text{s} .$$

29. If we write (for the general case) $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{v}$ is equal to

$$(yv_z - zv_y)\,\hat{\text{i}} + (zv_x - xv_z)\,\hat{\text{j}} + (xv_y - yv_x)\,\hat{\text{k}}\ .$$

(a) The angular momentum is given by the vector product $\vec{\ell} = m\vec{r} \times \vec{v}$, where $\vec{r}$ is the position vector of the particle, $\vec{v}$ is its velocity, and $m = 3.0$ kg is its mass. Substituting (with SI units understood) $x = 3$, $y = 8$, $z = 0$, $v_x = 5$, $v_y = -6$ and $v_z = 0$ into the above expression, we obtain

$$\vec{\ell} = (3.0)\,((3)(-6) - (8.0)(5.0))\,\hat{\text{k}} = -1.7 \times 10^2\,\hat{\text{k}}\ \text{kg}\cdot\text{m}^2/\text{s}\ .$$

(b) The torque is given by Eq. 12-14, $\vec{\tau} = \vec{r} \times \vec{F}$. We write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}}$ and $\vec{F} = F_x\,\hat{\text{i}}$ and obtain

$$\vec{\tau} = \left(x\,\hat{\text{i}} + y\,\hat{\text{j}}\right) \times \left(F_x\,\hat{\text{i}}\right) = -yF_x\,\hat{\text{k}}$$

since $\hat{\text{i}} \times \hat{\text{i}} = 0$ and $\hat{\text{j}} \times \hat{\text{i}} = -\hat{\text{k}}$. Thus, we find $\vec{\tau} = -(8.0\,\text{m})(-7.0\,\text{N})\,\hat{\text{k}} = 56\,\hat{\text{k}}\ \text{N}\cdot\text{m}$.

(c) According to Newton's second law $\vec{\tau} = d\vec{\ell}/dt$, so the rate of change of the angular momentum is $56\,\text{kg}\cdot\text{m}^2/\text{s}^2$, in the positive z direction.

44. Angular momentum conservation $I_i\omega_i = I_f\omega_f$ leads to

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}\omega_i = 3$$

which implies

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}I_f\omega_f^2}{\frac{1}{2}I_i\omega_i^2} = \frac{I_f}{I_i}\left(\frac{\omega_f}{\omega_i}\right)^2 = 3 \ .$$

69. We make the unconventional choice of *clockwise* sense as positive, so that the angular acceleration are positive (as is the linear acceleration of the center of mass, since we take rightwards as positive).

(a) We approach this in the manner of Eq. 12-3 (*pure rotation* about point P) but use torques instead of energy:

$$\tau = I_P\,\alpha \quad \text{where } I_P = \frac{1}{2}MR^2 + MR^2$$

where the parallel-axis theorem and Table 11-2(c) has been used. The torque (relative to point P) is due to the $F = 12$ N force and is $\tau = F(2R)$. In this way, we find

$$\alpha = \frac{(12)(0.20)}{0.05 + 0.10} = 16 \text{ rad/s}^2 \ .$$

Hence, $a_{\text{com}} = R\alpha = 1.6 \text{ m/s}^2$.

(b) As shown above, $\alpha = 16 \text{ rad/s}^2$.

(c) Applying Newton's second law in its linear form yields

$$(12 \text{ N}) - f = Ma_{\text{com}} \quad .$$

Therefore, $f = -4.0$ N. Contradicting what we assumed in setting up our force equation, the friction force is found to point *rightward* (with magnitude 4.0 N).