

CHAPTER 11

1. We find the spring constant from the compression caused by the increased weight:

$$k = mg/x = (65 \text{ kg})(9.80 \text{ m/s}^2)/(0.028 \text{ m}) = 2.28 \times 10^4 \text{ N/m.}$$

The frequency of vibration will be

$$f = (k/m)^{1/2}/2\pi = [(2.28 \times 10^4 \text{ N/m})/(1065 \text{ kg})]^{1/2}/2\pi = 0.74 \text{ Hz.}$$

2. We find the spring constant from the elongation caused by the increased weight:

$$k = \Delta mg/\Delta x = (80 \text{ N} - 55 \text{ N})/(0.85 \text{ m} - 0.65 \text{ m}) = 1.3 \times 10^2 \text{ N/m.}$$

3. In one period the particle will travel from one extreme position to the other (a distance of $2A$) and back again. The total distance traveled is

$$d = 4A = 4(0.25 \text{ m}) = 1.00 \text{ m.}$$

4. (a) We find the spring constant from the elongation caused by the weight:

$$k = mg/\Delta x = (2.7 \text{ kg})(9.80 \text{ m/s}^2)/(0.039 \text{ m}) = 6.8 \times 10^2 \text{ N/m.}$$

- (b) Because the fish will oscillate about the equilibrium position, the amplitude will be the distance the fish was pulled down from equilibrium:

$$A = 2.5 \text{ cm.}$$

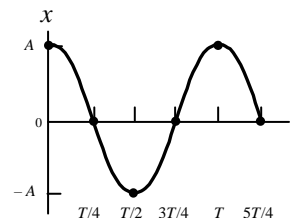
The frequency of vibration will be

$$f = (k/m)^{1/2}/2\pi = [(6.8 \times 10^2 \text{ N/m})/(2.7 \text{ kg})]^{1/2}/2\pi = 2.5 \text{ Hz.}$$

5. Because the mass starts at the maximum displacement, we have

t	0	$T/4$	$T/2$	$3T/4$	T	$5T/4$
x	A	0	$-A$	0	A	0

We see that the curve resembles a **cosine wave**.



6. (a) We find the effective spring constant from the frequency:

$$f_1 = (k/m_1)^{1/2}/2\pi;$$

$$4.0 \text{ Hz} = [k/(0.15 \times 10^{-3} \text{ kg})]^{1/2}/2\pi, \text{ which gives } k = 9.5 \times 10^{-2} \text{ N/m.}$$

- (b) The new frequency of vibration will be

$$f_2 = (k/m_2)^{1/2}/2\pi = [(9.5 \times 10^{-2} \text{ N/m})/(0.50 \times 10^{-3} \text{ kg})]^{1/2}/2\pi = 2.2 \text{ Hz.}$$

7. (a) We find the effective spring constant from the frequency:

$$f_1 = (k/m_1)^{1/2}/2\pi;$$

$$2.5 \text{ Hz} = [k/(0.050 \text{ kg})]^{1/2}/2\pi, \text{ which gives } k = 12 \text{ N/m.}$$

- (b) Because the size and shape are the same, the spring constant will be the same.

The new frequency of vibration will be

$$f_2 = (k/m_2)^{1/2}/2\pi = [(12 \text{ N/m})/(0.25 \text{ kg})]^{1/2}/2\pi = 1.1 \text{ Hz.}$$

8. The dependence of the frequency on the mass is

$$f = (k/m)^{1/2}/2\pi.$$

Because the spring constant does not change, we have

$$f_2/f_1 = (m_1/m_2)^{1/2};$$

$$f_2/(3.0 \text{ Hz}) = [(0.60 \text{ kg})/(0.38 \text{ kg})]^{1/2}, \text{ which gives } f_2 = 3.8 \text{ Hz}.$$

9. (a) The velocity will be maximum at the equilibrium position:

$$v_0 = A\omega = 2\pi fA = 2\pi(3.0 \text{ Hz})(0.15 \text{ m}) = 2.8 \text{ m/s}.$$

- (b) We find the velocity at the position from

$$v = v_0[1 - (x^2/A^2)]^{1/2}$$

$$= (2.8 \text{ m/s})\{1 - [(0.10 \text{ m})^2/(0.15 \text{ m})^2]\}^{1/2} = 2.1 \text{ m/s}.$$

- (c) We find the total energy from the maximum kinetic energy:

$$E = KE_{\text{max}} = \frac{1}{2}mv_0^2 = \frac{1}{2}(0.50 \text{ kg})(2.8 \text{ m/s})^2 = 2.0 \text{ J}.$$

- (d) Because
- $x = A$
- at
- $t = 0$
- , we have a cosine function:

$$x = A \cos(\omega t) = A \cos(2\pi ft) = (0.15 \text{ m}) \cos[2\pi(3.0 \text{ Hz})t].$$

10. The dependence of the frequency on the mass is

$$f = (k/m)^{1/2}/2\pi.$$

Because the spring constant does not change, we have

$$f_2/f_1 = (m/m_2)^{1/2};$$

$$(0.60 \text{ Hz})/(0.88 \text{ Hz}) = [m/(m + 0.600 \text{ kg})]^{1/2}, \text{ which gives } m = 0.52 \text{ kg}.$$

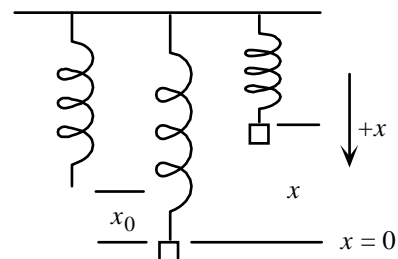
11. In the equilibrium position, the net force is zero. When the mass is pulled down a distance
- x
- , the net restoring force is the sum of the additional forces from the springs, so we have

$$F_{\text{net}} = \Delta F_2 + \Delta F_1 = -kx - kx = -2kx,$$

which gives an effective force constant of $2k$.

We find the frequency of vibration from

$$f = (k_{\text{eff}}/m)^{1/2}/2\pi = (2k/m)^{1/2}/2\pi.$$



12. We find the spring constant from the elongation caused by the mass:

$$k = \Delta mg / \Delta x = (1.62 \text{ kg})(9.80 \text{ m/s}^2)/(0.315 \text{ m}) = 50.4 \text{ N/m}.$$

The period of the motion is independent of amplitude:

$$T = 2\pi(m/k)^{1/2} = 2\pi[(1.62 \text{ kg})/(50.4 \text{ N/m})]^{1/2} = 1.13 \text{ s}.$$

The time to return to the equilibrium position is one-quarter of a period:

$$t = (T/4) = (1.13 \text{ s})/4 = 0.282 \text{ s}.$$

13. We find the spring constant from the compression caused by the force:

$$k = F / \Delta x = (80.0 \text{ N})/(0.200 \text{ m}) = 400 \text{ N/m}.$$

The ball will leave at the equilibrium position, where the kinetic energy is maximum. Because this is also the maximum potential energy, we have

$$KE_{\text{max}} = \frac{1}{2}mv_0^2 = PE_{\text{max}} = \frac{1}{2}kA^2;$$

$$\frac{1}{2}(0.150 \text{ kg})v_0^2 = \frac{1}{2}(400 \text{ N/m})(0.200 \text{ m})^2, \text{ which gives } v_0 = 10.3 \text{ m/s}.$$

14. (a) The period of the motion is independent of amplitude:
 $T = 2\pi(m/k)^{1/2} = 2\pi[(0.750 \text{ kg})/(124 \text{ N/m})]^{1/2} = 0.489 \text{ s}.$
 The frequency is
 $f = 1/T = 1/(0.489 \text{ s}) = 2.04 \text{ Hz}.$
- (b) Because the mass is struck at the equilibrium position, the initial speed is the maximum speed.
 We find the amplitude from
 $v_0 = A\omega = 2\pi fA;$
 $2.76 \text{ m/s} = 2\pi(2.04 \text{ Hz})A,$ which gives $A = 0.215 \text{ m}.$
- (c) The maximum acceleration is
 $a_{\text{max}} = \omega^2 A = (2\pi f)^2 A = [2\pi(2.04 \text{ Hz})]^2(0.215 \text{ m}) = 35.5 \text{ m/s}^2.$
- (d) Because the mass starts at the equilibrium position, we have a sine function. If we take the positive x -direction in the direction of the initial velocity, we have
 $x = A \sin(\omega t) = A \sin(2\pi ft) = (0.215 \text{ m}) \sin[2\pi(2.04 \text{ Hz})t] = (0.215 \text{ m}) \sin[(12.8 \text{ s}^{-1})t].$
- (e) We find the total energy from the maximum kinetic energy:
 $E = \text{KE}_{\text{max}} = \frac{1}{2}mv_0^2 = \frac{1}{2}(0.750 \text{ kg})(2.76 \text{ m/s})^2 = 2.86 \text{ J}.$
15. (a) The work done on the spring increases the potential energy:
 $W = \text{PE}_{\text{max}} = \frac{1}{2}kA_0^2;$
 $3.0 \text{ J} = \frac{1}{2}k(0.12 \text{ m})^2,$ which gives $k = 4.2 \times 10^2 \text{ N/m}.$
- (b) The maximum acceleration is produced by the maximum restoring force:
 $F_{\text{max}} = kA = ma;$
 $(4.17 \times 10^2 \text{ N/m})(0.12 \text{ m}) = m(15 \text{ m/s}^2),$ which gives $m = 3.3 \text{ kg}.$
16. Because the mass is released at the maximum displacement, we have
 $x = x_0 \cos(\omega t); \quad v = -v_0 \sin(\omega t); \quad a = -a_{\text{max}} \cos(\omega t).$
- (a) We find ωt from
 $v = -v_0 \sin(\omega t),$ which gives $\omega t = 30^\circ.$
 Thus the distance is
 $x = x_0 \cos(\omega t) = x_0 \cos(30^\circ) = 0.866 x_0.$
- (b) We find ωt from
 $a = -a_{\text{max}} \cos(\omega t),$ which gives $\omega t = 60^\circ.$
 Thus the distance is
 $x = x_0 \cos(\omega t) = x_0 \cos(60^\circ) = 0.500 x_0.$
17. (a) The amplitude is the maximum value of x : $0.45 \text{ m}.$
- (b) We find the frequency from the coefficient of t :
 $2\pi f = 8.40 \text{ s}^{-1},$ which gives $f = 1.34 \text{ Hz}.$
- (c) The maximum speed is
 $v_0 = \omega A = (8.40 \text{ s}^{-1})(0.45 \text{ m}) = 3.78 \text{ m/s}.$
 We find the total energy from the maximum kinetic energy:
 $E = \text{KE}_{\text{max}} = \frac{1}{2}mv_0^2 = \frac{1}{2}(0.50 \text{ kg})(3.78 \text{ m/s})^2 = 3.6 \text{ J}.$
- (d) We find the velocity at the position from
 $v = v_0[1 - (x^2/A^2)]^{1/2}$
 $= (3.78 \text{ m/s})\{1 - [(0.30 \text{ m})^2/(0.45 \text{ m})^2]\}^{1/2} = 2.82 \text{ m/s}.$
 The kinetic energy is
 $\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}(0.50 \text{ kg})(2.82 \text{ m/s})^2 = 2.0 \text{ J}.$
 The potential energy is

$$PE = E - KE = 3.6 \text{ J} - 2.0 \text{ J} = 1.6 \text{ J}.$$

18. (a) The amplitude is the maximum value of x : **0.35 m.**

(b) We find the frequency from the coefficient of t :

$$2\pi f = 5.50 \text{ s}^{-1}, \text{ which gives } f = \mathbf{0.875 \text{ Hz.}}$$

(c) The period is

$$T = 1/f = 1/(0.875 \text{ Hz}) = \mathbf{1.14 \text{ s.}}$$

(d) The maximum speed is

$$v_0 = \omega A = (5.50 \text{ s}^{-1})(0.35 \text{ m}) = 1.93 \text{ m/s.}$$

We find the total energy from the maximum kinetic energy:

$$E = KE_{\max} = \frac{1}{2}mv_0^2 = \frac{1}{2}(0.400 \text{ kg})(1.93 \text{ m/s})^2 = \mathbf{0.74 \text{ J.}}$$

(e) We find the velocity at the position from

$$v = v_0[1 - (x^2/A^2)]^{1/2}$$

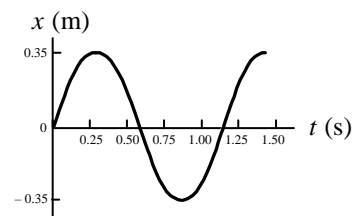
$$= (1.93 \text{ m/s})\{1 - [(0.10 \text{ m})^2/(0.35 \text{ m})^2]\}^{1/2} = 1.84 \text{ m/s.}$$

The kinetic energy is

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(0.400 \text{ kg})(1.84 \text{ m/s})^2 = \mathbf{0.68 \text{ J.}}$$

The potential energy is

$$PE = E - KE = 0.74 \text{ J} - 0.68 \text{ J} = \mathbf{0.06 \text{ J.}}$$



19. (a) We find the frequency from

$$f = (k/m)^{1/2}/2\pi = [(210 \text{ N/m})/(0.250 \text{ kg})]^{1/2}/2\pi = 4.61 \text{ Hz, so}$$

$$\omega = 2\pi f = 2\pi(4.61 \text{ Hz}) = 29.0 \text{ s}^{-1}.$$

Because the mass starts at the equilibrium position moving in the positive direction, we have a sine function:

$$x = A \sin(\omega t) = (0.280 \text{ m}) \sin[(29.0 \text{ s}^{-1})t].$$

(b) The period of the motion is

$$T = 1/f = 1/(4.61 \text{ Hz}) = 0.217 \text{ s.}$$

It will take one-quarter period to reach the maximum extension, so the spring will have maximum extensions at **0.0542 s, 0.271 s, 0.488 s, ...**

It will take three-quarters period to reach the minimum extension, so the spring will have minimum extensions at **0.163 s, 0.379 s, 0.596 s, ...**

20. (a) We find the frequency from the period:

$$f = 1/T = 1/(0.55 \text{ s}) = 1.82 \text{ Hz, so}$$

$$\omega = 2\pi f = 2\pi(1.82 \text{ Hz}) = 11.4 \text{ s}^{-1}.$$

The amplitude is the compression: 0.10 m. Because the mass is released at the maximum displacement, we have a cosine function:

$$y = A \cos(\omega t) = (0.10 \text{ m}) \cos[(11.4 \text{ s}^{-1})t].$$

(b) The time to return to the equilibrium position is one-quarter of a period:

$$t = (T/4) = (0.55 \text{ s})/4 = \mathbf{0.14 \text{ s.}}$$

(c) The maximum speed is

$$v_0 = \omega A = (11.4 \text{ s}^{-1})(0.10 \text{ m}) = \mathbf{1.1 \text{ m/s.}}$$

(d) The maximum acceleration is

$$a_{\max} = \omega^2 A = (11.4 \text{ s}^{-1})^2(0.10 \text{ m}) = \mathbf{13 \text{ m/s}^2}.$$

The maximum magnitude of the acceleration occurs at the endpoints of the motion, so it will be attained first at **the release point.**

21. Immediately after the collision, the block-bullet system will have its maximum velocity at the equilibrium position. We find this velocity from energy conservation:

$$KE_i + PE_i = KE_f + PE_f;$$

$$\frac{1}{2}(M + m)v_0^2 + 0 = 0 + \frac{1}{2}kA^2;$$

$$\frac{1}{2}(0.600 \text{ kg} + 0.025 \text{ kg})v_0^2 = \frac{1}{2}(6.70 \times 10^3 \text{ N/m})(0.215 \text{ m})^2, \text{ which gives } v_0 = 22.3 \text{ m/s}.$$

We find the initial speed of the bullet from momentum conservation for the impact:

$$mv + 0 = (M + m)v_0;$$

$$(0.025 \text{ kg})v = (0.600 \text{ kg} + 0.025 \text{ kg})(22.3 \text{ m/s}), \text{ which gives } v = \mathbf{557 \text{ m/s}}.$$

22. Because the frequencies and masses are the same, the spring constant must be the same. We can compare the two maximum potential energies:

$$PE_1/PE_2 = \frac{1}{2}kA_1^2 / \frac{1}{2}kA_2^2 = (A_1/A_2)^2;$$

$$10 = (A_1/A_2)^2, \text{ or } \mathbf{A_1 = 3.16A_2}.$$

23. (a) The total energy is the maximum potential energy, so we have

$$PE = \frac{1}{2}PE_{\max};$$

$$\frac{1}{2}kx^2 = \frac{1}{4}(kA^2), \text{ which gives } x = \mathbf{0.707A}.$$

- (b) We find the position from

$$v = v_0[1 - (x^2/A^2)]^{1/2};$$

$$\frac{1}{2}v_0 = v_0[1 - (x^2/A^2)]^{1/2}, \text{ which gives } x = \mathbf{0.866A}.$$

24. We use a coordinate system with down positive.

With x_0 a magnitude, at the equilibrium position we have

$$F = -kx_0 + mg = 0.$$

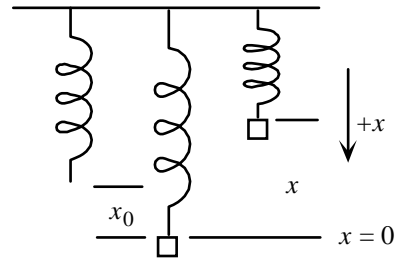
If the spring is compressed a distance x from the equilibrium position, we have

$$F = -k(x_0 + x) + mg = 0.$$

When we use the equilibrium condition, we get

$$F = F = -kx.$$

Note that x is negative, so the force is positive.



25. For the vertical spring there are both gravitational and elastic potential energy terms. We choose the unloaded position of the spring as the reference level with down positive.

With x_0 a magnitude, at the equilibrium position we have

$$F = -kx_0 + mg = 0, \text{ or } kx_0 = mg.$$

When the spring is stretched a distance x from the equilibrium position, the stretch of the spring is $x + x_0$.

For the total energy we have

$$E = KE + PE_{\text{grav}} + PE_{\text{spring}}$$

$$= \frac{1}{2}mv^2 - mg(x + x_0) + \frac{1}{2}k(x + x_0)^2 = \frac{1}{2}mv^2 - mgx - mgx_0 + \frac{1}{2}kx^2 + kxx_0 + \frac{1}{2}kx_0^2.$$

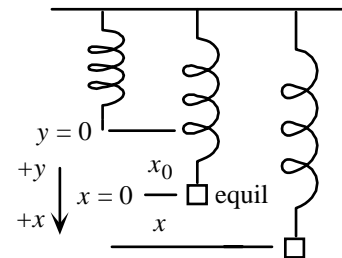
When we use the equilibrium condition, we get

$$E = \frac{1}{2}mv^2 - mgx_0 + \frac{1}{2}kx^2 + \frac{1}{2}kx_0^2.$$

Because the terms containing x_0 are constant, we have

$$E + mgx_0 - \frac{1}{2}kx_0^2 = (\text{a constant}) = E' = \frac{1}{2}mv^2 + \frac{1}{2}kx^2.$$

If we had chosen a reference level for the gravitational potential energy halfway between the unloaded



position and the equilibrium position, the terms added to E would cancel and $E = E'$.

26. We find the period from the time for N oscillations:

$$T = t/N = (34.7 \text{ s})/8 = 4.34 \text{ s}.$$

From this we can get the spring constant:

$$T = 2\pi(m/k)^{1/2};$$

$$4.34 \text{ s} = 2\pi[(65.0 \text{ kg})/k]^{1/2}, \text{ which gives } k = \quad \mathbf{136 \text{ N/m}}.$$

At the equilibrium position, we have

$$mg = kx_0;$$

$$(65.0 \text{ kg})(9.80 \text{ m/s}^2) = (136 \text{ N/m})x_0, \text{ which gives } x_0 = 4.7 \text{ m}.$$

Because this is how much the cord has stretched, we have

$$L = D - x_0 = 25.0 \text{ m} - 4.7 \text{ m} = \quad \mathbf{20.3 \text{ m}}.$$

27. (a) If we apply a force F to stretch the springs, the total displacement Δx is the sum of the displacements of the two springs: $\Delta x = \Delta x_1 + \Delta x_2$.

The effective spring constant is defined from $F = -k_{\text{eff}}\Delta x$.

Because they are in series, the force must be the same in each spring:

$$F_1 = F_2 = F = -k_1\Delta x_1 = -k_2\Delta x_2.$$

Then $\Delta x = \Delta x_1 + \Delta x_2$ becomes

$$-F/k_{\text{eff}} = -(F/k_1) - (F/k_2), \quad \text{or} \quad 1/k_{\text{eff}} = (1/k_1) + (1/k_2).$$

For the period we have

$$T = 2\pi(m/k_{\text{eff}})^{1/2} = 2\pi\{m[(1/k_1) + (1/k_2)]\}^{1/2}.$$

- (b) In the equilibrium position, we have

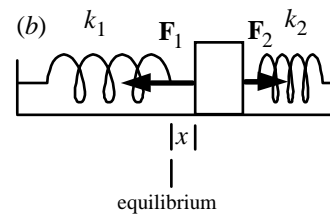
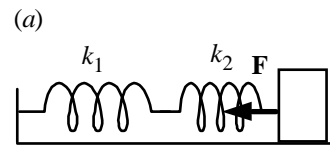
$$F_{\text{net}} = F_{20} - F_{10} = 0, \quad \text{or} \quad F_{10} = F_{20}.$$

When the object is moved to the right a distance x , we have

$$F_{\text{net}} = F_{20} - k_2x - (F_{10} + k_1x) = -(k_1 + k_2)x.$$

The effective spring constant is $k_{\text{eff}} = k_1 + k_2$, so the period is

$$T = 2\pi(m/k_{\text{eff}})^{1/2} = 2\pi[m/(k_1 + k_2)]^{1/2}.$$



28. (a) We find the period from the time for N oscillations:

$$T = t/N = (50 \text{ s})/36 = \quad \mathbf{1.4 \text{ s}}.$$

- (b) The frequency is

$$f = 1/T = 1/(1.39 \text{ s}) = \quad \mathbf{0.72 \text{ Hz}}.$$

29. (a) Because the period includes one "tick" and one "tock", the period is two seconds. We find the length from

$$T = 2\pi(L/g)^{1/2};$$

$$2.00 \text{ s} = 2\pi[(L/(9.80 \text{ m/s}^2))]^{1/2}, \text{ which gives } L = \quad \mathbf{0.993 \text{ m}}.$$

- (b) We see that an increase in L will cause an increase in T . This means that there will be fewer swings each hour, so the clock will run **slow**.

30. (a) For the period on Earth we have

$$T = 2\pi(L/g)^{1/2} = 2\pi[(0.50 \text{ m})/(9.80 \text{ m/s}^2)]^{1/2} = \quad \mathbf{1.4 \text{ s}}.$$

- (b) In a freely falling elevator, the effective g is zero, so the period would be **infinite (no swing)**.

31. (a) For the frequency we have

$$f = (g/L)^{1/2}(1/2\pi) = [(9.80 \text{ m/s}^2)/(0.66 \text{ m})]^{1/2}(1/2\pi) = 0.61 \text{ Hz.}$$

- (b) We use energy conservation between the release point and the lowest point:

$$KE_i + PE_i = KE_f + PE_f;$$

$$0 + mgh = \frac{1}{2}mv_0^2 + 0;$$

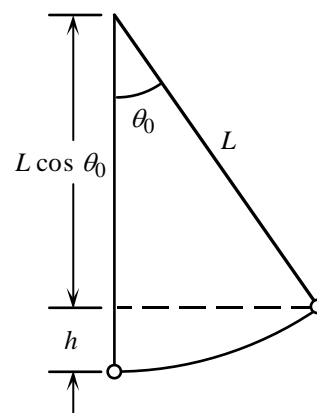
$$(9.80 \text{ m/s}^2)(0.66 \text{ m})(1 - \cos 12^\circ) = \frac{1}{2}v_0^2,$$

which gives $v_0 = 0.53 \text{ m/s}$.

- (c) The energy stored in the oscillation is the initial potential energy:

$$PE_i = mgh = (0.310 \text{ kg})(9.80 \text{ m/s}^2)(0.66 \text{ m})(1 - \cos 12^\circ) = 0.044 \text{ J.}$$

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32. We use energy conservation between the release point and the lowest point:

$$KE_i + PE_i = KE_f + PE_f;$$

$$0 + mgh = \frac{1}{2}mv_0^2 + 0, \text{ or } v_0^2 = 2gh = 2gL(1 - \cos \theta_0).$$

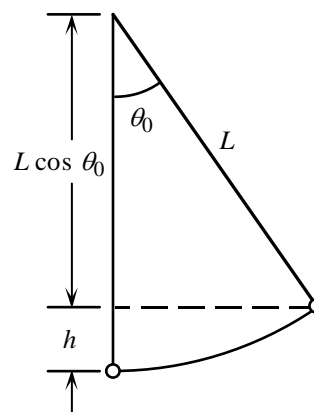
When we use a trigonometric identity, we get

$$v_0^2 = 2gL(2 \sin^2 \frac{1}{2}\theta_0).$$

For a simple pendulum θ_0 is small, so we have $\sin \frac{1}{2}\theta_0 \approx \frac{1}{2}\theta_0$.

Thus we get

$$v_0^2 = 2gL2 \left(\frac{1}{2}\theta_0\right)^2, \text{ or } v_0 = \theta_0(gL)^{1/2}.$$



33. We assume that
- 15°
- is small enough that we can consider this a simple pendulum, with a period

$$T = 1/f = 1/(2.0 \text{ Hz}) = 0.50 \text{ s.}$$

Because the pendulum is released at the maximum angle, the angle will oscillate as a cosine function:

$$\theta = \theta_0 \cos (2\pi ft) = (15^\circ) \cos [2\pi(2.0 \text{ Hz})t] = (15^\circ) \cos [(4.0\pi \text{ s}^{-1})t].$$

- (a)
- $\theta = (15^\circ) \cos [2\pi(2.0 \text{ Hz})(0.25 \text{ s})] = -15^\circ.$

This is expected, since the time is half a period.

- (b)
- $\theta = (15^\circ) \cos [2\pi(2.0 \text{ Hz})(1.60 \text{ s})] = +4.6^\circ.$

- (c)
- $\theta = (15^\circ) \cos [2\pi(2.0 \text{ Hz})(500 \text{ s})] = +15^\circ.$

This is expected, since the time is one thousand periods.

34. The speed of the wave is

$$v = f\lambda = \lambda/T = (8.5 \text{ m})/(3.0 \text{ s}) = 2.8 \text{ m/s.}$$

35. We find the wavelength from

$$v = f\lambda;$$

$$330 \text{ m/s} = (262 \text{ Hz})\lambda, \text{ which gives } \lambda = 1.26 \text{ m.}$$

36. For AM we find the wavelengths from

$$\lambda_{\text{AMhigher}} = v/f_{\text{AMlower}} = (3.00 \times 10^8 \text{ m/s}) / (550 \times 10^3 \text{ Hz}) = 545 \text{ m};$$

$$\lambda_{\text{AMlower}} = v/f_{\text{AMhigher}} = (3.00 \times 10^8 \text{ m/s}) / (1600 \times 10^3 \text{ Hz}) = 188 \text{ m}.$$

For FM we have

$$\lambda_{\text{FMhigher}} = v/f_{\text{FMlower}} = (3.00 \times 10^8 \text{ m/s}) / (88.0 \times 10^6 \text{ Hz}) = 3.41 \text{ m};$$

$$\lambda_{\text{FMlower}} = v/f_{\text{FMhigher}} = (3.00 \times 10^8 \text{ m/s}) / (108 \times 10^6 \text{ Hz}) = 2.78 \text{ m}.$$

37. We find the speed of the longitudinal (compression) wave from

$$v = (B/\rho)^{1/2} \text{ for fluids and } v = (E/\rho)^{1/2} \text{ for solids.}$$

(a) For water we have

$$v = (B/\rho)^{1/2} = [(2.0 \times 10^9 \text{ N/m}^2) / (1.00 \times 10^3 \text{ kg/m}^3)]^{1/2} = 1.4 \times 10^3 \text{ m/s}.$$

(b) For granite we have

$$v = (E/\rho)^{1/2} = [(45 \times 10^9 \text{ N/m}^2) / (2.7 \times 10^3 \text{ kg/m}^3)]^{1/2} = 4.1 \times 10^3 \text{ m/s}.$$

(c) For steel we have

$$v = (E/\rho)^{1/2} = [(200 \times 10^9 \text{ N/m}^2) / (7.8 \times 10^3 \text{ kg/m}^3)]^{1/2} = 5.1 \times 10^3 \text{ m/s}.$$

38. Because the modulus does not change, the speed depends on the density:

$$v \propto (1/\rho)^{1/2}.$$

Thus we see that the speed will be greater in the **less dense rod**.

For the ratio of speeds we have

$$v_1/v_2 = (\rho_2/\rho_1)^{1/2} = (2)^{1/2} = 1.41.$$

39. We find the speed of the wave from

$$v = [F_T/(m/L)]^{1/2} = \{(150 \text{ N}) / [(0.55 \text{ kg}) / (30 \text{ m})]\}^{1/2} = 90.5 \text{ m/s}.$$

We find the time from

$$t = L/v = (30 \text{ m}) / (90.5 \text{ m/s}) = 0.33 \text{ s}.$$

40. The speed of the longitudinal (compression) wave is

$$v = (E/\rho)^{1/2},$$

so the wavelength is

$$\lambda = v/f = (E/\rho)^{1/2}/f = [(100 \times 10^9 \text{ N/m}^2) / (7.8 \times 10^3 \text{ kg/m}^3)]^{1/2} / (6,000 \text{ Hz}) = 0.60 \text{ m}.$$

41. The speed of the longitudinal wave is

$$v = (B/\rho)^{1/2},$$

so the distance that the wave traveled is

$$2D = vt = (B/\rho)^{1/2}t;$$

$$2D = [(2.0 \times 10^9 \text{ N/m}^2) / (1.00 \times 10^3 \text{ kg/m}^3)]^{1/2}(3.0 \text{ s}), \text{ which gives } D = 2.1 \times 10^3 \text{ m} = 2.1 \text{ km}.$$

42. (a) Because both waves travel the same distance, we have

$$\mathcal{A}t = (d/v_S) - (d/v_P) = d[(1/v_S) - (1/v_P)];$$

$$(2.0 \text{ min})(60 \text{ s/min}) = d\{[1/(5.5 \text{ km/s})] - [1/(8.5 \text{ km/s})]\}, \text{ which gives } d = 1.9 \times 10^3 \text{ km}.$$

(b) The direction of the waves is not known, thus the position of the epicenter **cannot be determined**.

It would take at least one more station to find the intersection of the two circles.

43. For the surface wave we have

$$\omega = 2\pi f = 2\pi(0.50 \text{ Hz}) = 1 \text{ s}^{-1}.$$

The object will leave the surface when the maximum acceleration of the SHM becomes greater than g , so the normal force becomes zero. Thus we have

$$a_{\text{max}} = \omega^2 A > g;$$

$$(1 \text{ s}^{-1})^2 A > 9.80 \text{ m/s}^2, \text{ which gives } A > \quad 1.0 \text{ m}.$$

44. We assume that the wave spreads out uniformly in all directions.

- (a) The intensity will decrease as
- $1/r^2$
- , so the ratio of intensities is

$$I_2/I_1 = (r_1/r_2)^2 = [(10 \text{ km})/(20 \text{ km})]^2 = \quad 0.25.$$

- (b) Because the intensity depends on
- A^2
- , the amplitude will decrease as
- $1/r$
- , so the ratio of amplitudes is

$$A_2/A_1 = (r_1/r_2) = [(10 \text{ km})/(20 \text{ km})] = \quad 0.50.$$

45. We assume that the wave spreads out uniformly in all directions.

- (a) The intensity will decrease as
- $1/r^2$
- , so the ratio of intensities is

$$I_2/I_1 = (r_1/r_2)^2;$$

$$I_2/(2.0 \times 10^6 \text{ J/m}^2 \cdot \text{s}) = [(50 \text{ km})/(1.0 \text{ km})]^2, \text{ which gives } I_2 = \quad 5.0 \times 10^9 \text{ J/m}^2 \cdot \text{s}.$$

- (b) We can take the intensity to be constant over the small area, so we have

$$P_2 = I_2 S = (5.0 \times 10^9 \text{ J/m}^2 \cdot \text{s})(10.0 \text{ m}^2) = \quad 5.0 \times 10^{10} \text{ W}.$$

46. If we consider two concentric circles around the spot where the waves are generated, the same energy must go past each circle in the same time. The intensity of a wave depends on
- A^2
- , so for the energy passing through a circle of radius
- r
- , we have

$$E = I(2\pi r) = kA^2 2\pi r = \text{a constant}.$$

Thus A must vary with r , in particular, we have $A \propto 1/\sqrt{r}$.

47. Because the speed and frequency are the same for the two waves, the intensity depends on the amplitude:

$$I \propto A^2.$$

For the ratio of intensities we have

$$I_2/I_1 = (A_2/A_1)^2;$$

$$2 = (A_2/A_1)^2, \text{ which gives } A_2/A_1 = \quad 1.41.$$

48. Because the speed and frequency are the same for the two waves, the intensity depends on the amplitude:

$$I \propto A^2.$$

For the ratio of intensities we have

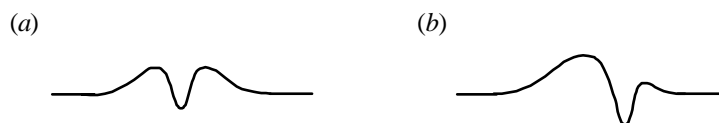
$$I_2/I_1 = (A_2/A_1)^2;$$

$$3 = (A_2/A_1)^2, \text{ which gives } A_2/A_1 = \quad 1.73.$$

49. The bug will undergo SHM, so the maximum KE is also the maximum PE, which occurs at the maximum displacement. For the ratio of energies we have

$$KE_2/KE_1 = PE_2/PE_1 = (A_2/A_1)^2 = [(4.5 \text{ cm})/(6.0 \text{ cm})]^2 = \quad 0.56.$$

50.



- (c) Because all particles of the string are at equilibrium positions, there is no potential energy. Particles of the string will have transverse velocities, so they have **kinetic energy**.

51. All harmonics are present in a vibrating string. Because the harmonic specifies the multiple of the fundamental, we have $f_n = nf_1$, $n = 1, 2, 3, \dots$:

$$f_1 = 1f_1 = (1)(440 \text{ Hz}) = 440 \text{ Hz};$$

$$f_2 = 2f_1 = (2)(440 \text{ Hz}) = 880 \text{ Hz};$$

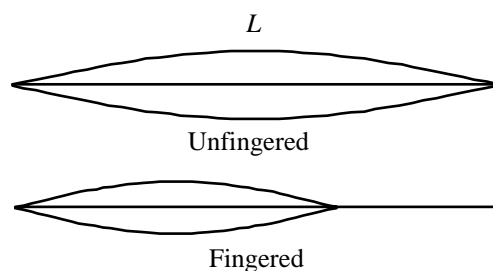
$$f_3 = 3f_1 = (3)(440 \text{ Hz}) = 1320 \text{ Hz};$$

$$f_4 = 4f_1 = (4)(440 \text{ Hz}) = 1760 \text{ Hz}.$$

52. From the diagram the initial wavelength is $2L$, and the final wavelength is $4L/3$. The tension has not changed, so the velocity has not changed:

$$v = f_1\lambda_1 = f_2\lambda_2;$$

$$(294 \text{ Hz})(2L) = f_2(4L/3), \text{ which gives } f_2 = 441 \text{ Hz}.$$



53. From the diagram the initial wavelength is $L/2$.

We see that the other wavelengths are

$$\lambda_1 = 2L, \lambda_2 = L \text{ and } \lambda_3 = 2L/3.$$

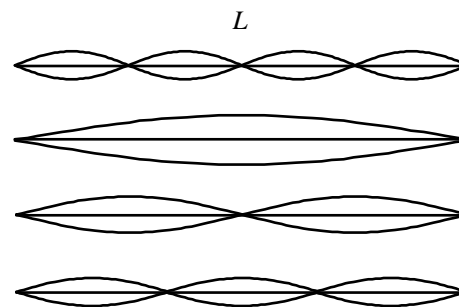
The tension has not changed, so the velocity has not changed:

$$v = f\lambda = f_n\lambda_n;$$

$$(280 \text{ Hz})(L/2) = f_1(2L), \text{ which gives } f_1 = 70 \text{ Hz};$$

$$(280 \text{ Hz})(L/2) = f_2(L), \text{ which gives } f_2 = 140 \text{ Hz};$$

$$(280 \text{ Hz})(L/2) = f_3(2L/3), \text{ which gives } f_3 = 210 \text{ Hz}.$$



54. The oscillation corresponds to the fundamental with a frequency:

$$f_1 = 1/T = 1/(2.5 \text{ s}) = 0.40 \text{ Hz}.$$

This is similar to the vibrating string, so all harmonics are present:

$$f_n = nf_1 = n(0.40 \text{ Hz}), n = 1, 2, 3, \dots$$

We find the corresponding periods from

$$T_n = 1/f_n = 1/nf_1 = T/n = (2.5 \text{ s})/n, n = 1, 2, 3, \dots$$

55. We find the wavelength from

$$v = f\lambda;$$

$$92 \text{ m/s} = (475 \text{ Hz})\lambda, \text{ which gives } \lambda = 0.194 \text{ m}.$$

The distance between adjacent nodes is $\lambda/2$, so we have

$$d = \lambda/2 = (0.194 \text{ m})/2 = 0.097 \text{ m}.$$

56. All harmonics are present in a vibrating string: $f_n = nf_1$, $n = 1, 2, 3, \dots$. The difference in frequencies for two successive overtones is

$$\Delta f = f_{n+1} - f_n = (n+1)f_1 - nf_1 = f_1, \text{ so we have } f_1 = 350 \text{ Hz} - 280 \text{ Hz} = \mathbf{70 \text{ Hz}}.$$

57. We find the speed of the wave from

$$v = [F_T / (m/L_0)]^{1/2} = \{(520 \text{ N}) / [(0.0036 \text{ kg}) / (0.90 \text{ m})]\}^{1/2} = 361 \text{ m/s}.$$

The wavelength of the fundamental for a string is $\lambda_1 = 2L$. We find the fundamental frequency from

$$f_1 = v / \lambda_1 = (361 \text{ m/s}) / 2(0.60 \text{ m}) = \mathbf{300 \text{ Hz}}.$$

All harmonics are present so the first overtone is the second harmonic:

$$f_2 = (2)f_1 = (2)300 \text{ Hz} = \mathbf{600 \text{ Hz}}.$$

The second overtone is the third harmonic:

$$f_3 = (3)f_1 = (3)300 \text{ Hz} = \mathbf{900 \text{ Hz}}.$$

58. We assume that the change in tension does not change the mass density, so the velocity variation depends only on the tension. Because the wavelength does not change, we have

$$\lambda = v_1 / f_1 = v_2 / f_2, \text{ or } F_{T2} / F_{T1} = (f_2 / f_1)^2.$$

For the fractional change we have

$$(F_{T2} - F_{T1}) / F_{T1} = (F_{T2} / F_{T1}) - 1 = (f_2 / f_1)^2 - 1 = [(200 \text{ Hz}) / (205 \text{ Hz})]^2 - 1 = -0.048.$$

Thus the tension should be **decreased by 4.8%**.

59. The speed of the wave depends on the tension and the mass density:

$$v = (F_T / \mu)^{1/2}.$$

The wavelength of the fundamental for a string is $\lambda_1 = 2L$. We find the fundamental frequency from

$$f_1 = v / \lambda_1 = (1/2L)(F_T / \mu)^{1/2}.$$

All harmonics are present in a vibrating string, so we have

$$f_n = nf_1 = (n/2L)(F_T / \mu)^{1/2}, n = 1, 2, 3, \dots$$

60. The hanging weight creates the tension in the string: $F_T = mg$. The speed of the wave depends on the tension and the mass density:

$$v = (F_T / \mu)^{1/2} = (mg / \mu)^{1/2}.$$

The frequency is fixed by the vibrator, so the wavelength is

$$\lambda = v / f = (1/f)(mg / \mu)^{1/2}. \text{ With a node at each end, each loop corresponds to } \lambda/2.$$

- (a) For one loop, we have $\lambda_1/2 = L$, or

$$2L = v_1 / f = (1/f)(m_1 g / \mu)^{1/2};$$

$$2(1.50 \text{ m}) = (1/60 \text{ Hz})[m_1(9.80 \text{ m/s}^2) / (4.3 \times 10^{-4} \text{ kg/m})]^{1/2}, \text{ which gives } m_1 = \mathbf{1.4 \text{ kg}}.$$

- (b) For two loops, we have $\lambda_2/2 = L/2$, or

$$L = v_2 / f = (1/f)(m_2 g / \mu)^{1/2};$$

$$1.50 \text{ m} = (1/60 \text{ Hz})[m_2(9.80 \text{ m/s}^2) / (4.3 \times 10^{-4} \text{ kg/m})]^{1/2}, \text{ which gives } m_2 = \mathbf{0.36 \text{ kg}}.$$

- (c) For five loops, we have $\lambda_5/2 = L/5$, or

$$2L/5 = v_5 / f = (1/f)(m_5 g / \mu)^{1/2};$$

$$2(1.50 \text{ m})/5 = (1/60 \text{ Hz})[m_5(9.80 \text{ m/s}^2) / (4.3 \times 10^{-4} \text{ kg/m})]^{1/2}, \text{ which gives } m_5 = \mathbf{0.057 \text{ kg}}.$$

The amplitude of the standing wave can be much greater than the vibrator amplitude because of the resonance built up from the reflected waves at the two ends of the string.

61. The hanging weight creates the tension in the string: $F_T = mg$. The speed of the wave depends on the tension and the mass density:

$$v = (F_T/\mu)^{1/2} = (mg/\mu)^{1/2}, \text{ and thus is constant.}$$

The frequency is fixed by the vibrator, so the constant wavelength is

$$\lambda = v/f = (1/f)(mg/\mu)^{1/2} = (1/60 \text{ Hz})[(0.080 \text{ kg})(9.80 \text{ m/s}^2)/(5.6 \times 10^{-4} \text{ kg/m})]^{1/2} = 0.624 \text{ m.}$$

The different standing waves correspond to different integral numbers of loops, starting at one loop.

With a node at each end, each loop corresponds to $\lambda/2$. The lengths of the string for the possible standing wavelengths are

$$L_n = n\lambda/2 = n(0.624 \text{ m})/2 = n(0.312 \text{ m}), n = 1, 2, 3, \dots, \text{ or}$$

$$L_n = 0.312 \text{ m}, 0.624 \text{ m}, 0.924 \text{ m}, 1.248 \text{ m}, 1.560 \text{ m}, \dots$$

Thus we see that there are **4** standing waves for lengths between 0.10 m and 1.5 m.

62. (a) The wavelength of the fundamental for a string is $2L$, so the fundamental frequency is

$$f = (1/2L)(F_T/\mu)^{1/2}.$$

When the tension is changed, the change in frequency is

$$\begin{aligned} \Delta f &= f' - f = (1/2L)[(F_T'/\mu)^{1/2} - (F_T/\mu)^{1/2}] \\ &= (1/2L)\{(F_T/\mu)^{1/2}[(F_T'/F_T)^{1/2} - 1]\} = f\{[(F_T + \Delta F_T)/F_T]^{1/2} - 1\} \\ &= f\{[1 + (\Delta F_T/F_T)]^{1/2} - 1\}. \end{aligned}$$

If $\Delta F_T/F_T$ is small, we have

$$[1 + (\Delta F_T/F_T)]^{1/2} \approx 1 + \frac{1}{2}(\Delta F_T/F_T), \text{ so we get}$$

$$\Delta f \approx f[1 + \frac{1}{2}(\Delta F_T/F_T) - 1] = \frac{1}{2}(\Delta F_T/F_T)f.$$

- (b) With the given data, we get

$$\Delta f = \frac{1}{2}(\Delta F_T/F_T)f;$$

$$442 \text{ Hz} - 438 \text{ Hz} = \frac{1}{2}(\Delta F_T/F_T)(438 \text{ Hz}), \text{ which gives } \Delta F_T/F_T = 0.018 = \mathbf{1.8\% \text{ (increase)}}.$$

- (c) For each overtone there will be a new wavelength, but the wavelength does not change when the tension changes, so the formula **will apply** to the overtones.

63. For the refraction of the waves we have

$$v_2/v_1 = (\sin \theta_2)/(\sin \theta_1);$$

$$v_2/(8.0 \text{ km/s}) = (\sin 31^\circ)/(\sin 50^\circ), \text{ which gives } v_2 = \mathbf{5.4 \text{ km/s}}.$$

64. For the refraction of the waves we have

$$v_2/v_1 = (\sin \theta_2)/(\sin \theta_1);$$

$$(2.1 \text{ km/s})/(2.8 \text{ km/s}) = (\sin \theta_2)/(\sin 34^\circ), \text{ which gives } \theta_2 = \mathbf{25^\circ}.$$

65. The speed of the longitudinal (compression) wave for the solid rock depends on the modulus and the density: $v = (E/\rho)^{1/2}$. The modulus does not change, so we have $v \propto (1/\rho)^{1/2}$.

For the refraction of the waves we have

$$v_2/v_1 = (\rho_1/\rho_2)^{1/2} = (SG_1/SG_2)^{1/2} = (\sin \theta_2)/(\sin \theta_1);$$

$$[(3.7)/(2.8)]^{1/2} = (\sin \theta_2)/(\sin 35^\circ), \text{ which gives } \theta_2 = \mathbf{41^\circ}.$$

66. (a) For the refraction of the waves we have

$$v_2/v_1 = (\sin \theta_2)/(\sin \theta_1);$$

Because $v_2 > v_1$, $\theta_2 > \theta_1$. When we use the maximum value of θ_2 , we get

$$v_2/v_1 = (\sin 90^\circ)/(\sin \theta_{1M}) = 1/(\sin \theta_{1M}), \text{ or } \mathbf{\theta_{1M} = \sin^{-1}(v_1/v_2)}.$$

(b) We have

$$\theta_M = \sin^{-1}(v_1/v_2) = \sin^{-1}[(7.2 \text{ km/s})/(8.4 \text{ km/s})] = 59^\circ.$$

Thus for angles $> 59^\circ$ there will be only reflection.

67. If we approximate the sloshing as a standing wave with the fundamental frequency, we have $\lambda = 2D$. We find the speed of the waves from

$$v = f\lambda = (1.0 \text{ Hz})2(0.08 \text{ m}) = \mathbf{0.16 \text{ m/s.}}$$

68. We choose $h = 0$ at the unstretched position of the net and let the stretch of the net be x . We use energy conservation between the release point and the lowest point to find the spring constant:

$$KE_i + PE_i = KE_f + PE_f;$$

$$0 + mgh_i = 0 + mgh_f + !kx_1^2, \text{ or } mg[h_i - (-x_1)] = !kx_1^2;$$

$$(70 \text{ kg})(9.80 \text{ m/s}^2)(20 \text{ m} + 1.1 \text{ m}) = !k(1.1 \text{ m})^2, \text{ which gives } k = 2.39 \times 10^4 \text{ N/m.}$$

When the person lies on the net, the weight causes the deflection:

$$mg = kx_2;$$

$$(70 \text{ kg})(9.80 \text{ m/s}^2) = (2.39 \times 10^4 \text{ N/m})x_2, \text{ which gives } x_2 = 0.029 \text{ m} = \mathbf{2.9 \text{ cm.}}$$

We use energy conservation between the release point and the lowest point to find the stretch:

$$KE_i + PE_i = KE_f + PE_f;$$

$$0 + mgh_i = 0 + mgh_f + !kx_3^2, \text{ or } mg[h_i - (-x_3)] = !kx_3^2;$$

$$(70 \text{ kg})(9.80 \text{ m/s}^2)(35 \text{ m} + x_3) = !(2.39 \times 10^4 \text{ N/m})x_3^2.$$

This is a quadratic equation for x_3 , for which the positive result is $\mathbf{1.4 \text{ m.}}$

69. The stress from the tension in the cable causes the strain. We find the effective spring constant from

$$E = \text{stress/strain} = (F_T/A)/(\Delta L/L_0), \text{ or}$$

$$k = F_T/\Delta L = EA/L_0 = (200 \times 10^9 \text{ N/m}^2)(3.2 \times 10^{-3} \text{ m})^2/(20 \text{ m}) = 3.22 \times 10^5 \text{ N/m.}$$

We find the period from

$$T = 2\pi(m/k)^{1/2} = 2\pi[(1200 \text{ kg})/(3.22 \times 10^5 \text{ N/m})]^{1/2} = \mathbf{0.38 \text{ s.}}$$

70. We ignore any frictional losses and use energy conservation:

$$KE_i + PE_i = KE_f + PE_f;$$

$$!mv_0^2 + 0 = 0 + !kx^2;$$

$$!(1500 \text{ kg})(2 \text{ m/s})^2 = !(500 \times 10^3 \text{ N/m})x^2, \text{ which gives } x = 0.11 \text{ m} = \mathbf{11 \text{ cm.}}$$

71. We treat the oscillation of the Jell-O as a standing wave produced by shear waves traveling up and down. The speed of the shear waves is

$$v = (G/\rho)^{1/2} = [(520 \text{ N/m}^2)/(1300 \text{ kg/m}^3)]^{1/2} = 0.632 \text{ m/s.}$$

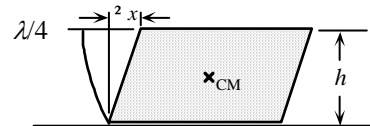
Because the maximum shear displacement is at the top, we estimate the wavelength as

$$\lambda = 4h = 4(0.040 \text{ m}) = 0.16 \text{ m.}$$

We find the frequency from

$$v = f\lambda;$$

$$0.632 \text{ m/s} = f(0.16 \text{ m}), \text{ which gives } f = \mathbf{4.0 \text{ Hz.}}$$



72. The effective value of g is increased when the acceleration is upward and decreased when the acceleration is downward. Because the length does not change, for the ratio of frequencies we have

$$f'/f = (g'/g)^{1/2}.$$

(a) For the upward acceleration we get

$$f'/f = (g'/g)^{1/2} = [(g + !g)/g]^{1/2}, \text{ which gives } f' = \mathbf{1.22f.}$$

(b) For the downward acceleration we get

$$f' / f = (g' / g)^{1/2} = [(g - \Delta g) / g]^{1/2}, \text{ which gives } f' = 0.71f.$$

73. (a) We find the effective force constant from the displacement caused by the additional weight:

$$k \Delta y = mg, \text{ or } k = \Delta mg / \Delta y = (75 \text{ kg})(9.80 \text{ m/s}^2) / (0.040 \text{ m}) = 1.84 \times 10^4 \text{ N/m.}$$

We find the frequency of vibration from

$$f = (k/m)^{1/2}(1/2\pi) = [(1.84 \times 10^4 \text{ N/m}) / (250 \text{ kg})]^{1/2}(1/2\pi) = 1.4 \text{ Hz.}$$

- (b) The total energy is the maximum potential energy, so we have

$$E = PE_{\text{max}} = \frac{1}{2}kA^2 = \frac{1}{2}(1.84 \times 10^4 \text{ N/m})(0.040 \text{ m})^2 = 15 \text{ J.}$$

Note that this is similar to the weight hanging on a spring. If we measure from the equilibrium position, we ignore changes in mgh .

74. The frequency of the sound will be the frequency of the needle passing over the ripples. The speed of the needle relative to the ripples is $v = r\omega$, so the frequency is

$$f = v/\lambda = r\omega/\lambda \\ = (0.128 \text{ m})(33 \text{ rev/min})(2\pi \text{ rad/rev}) / (60 \text{ s/min})(1.70 \times 10^{-3} \text{ m}) = 260 \text{ Hz.}$$

75. (a) All harmonics are present in a vibrating string: $f_n = nf_1$, $n = 1, 2, 3, \dots$

The first overtone is f_2 and the second overtone is f_3 .

For G we have

$$f_2 = 2(392 \text{ Hz}) = 784 \text{ Hz}; \quad f_3 = 3(392 \text{ Hz}) = 1176 \text{ Hz.}$$

For A we have

$$f_2 = 2(440 \text{ Hz}) = 880 \text{ Hz}; \quad f_3 = 3(440 \text{ Hz}) = 1320 \text{ Hz.}$$

- (b) The speed of the wave in a string is $v = [F_T/(M/L)]^{1/2}$. Because the lengths are the same, the wavelengths of the fundamentals must be the same. For the ratio of frequencies we have

$$f_A/f_G = v_A/v_G = (M_G/M_A)^{1/2};$$

$$(440 \text{ Hz}) / (392 \text{ Hz}) = (M_G/M_A)^{1/2}, \text{ which gives } M_G/M_A = 1.26.$$

- (c) Because the mass densities and the tensions are the same, the speeds must be the same. The wavelengths are proportional to the lengths, so for the ratio of frequencies we have

$$f_A/f_G = \lambda_G/\lambda_A = L_G/L_A;$$

$$(440 \text{ Hz}) / (392 \text{ Hz}) = L_G/L_A, \text{ which gives } L_G/L_A = 1.12.$$

- (d) The speed of the wave in a string is $v = [F_T/(M/L)]^{1/2}$. Because the lengths are the same, the wavelengths of the fundamentals must be the same. For the ratio of frequencies we have

$$f_A/f_G = v_A/v_G = (F_{TA}/F_{TG})^{1/2};$$

$$(440 \text{ Hz}) / (392 \text{ Hz}) = (F_{TA}/F_{TG})^{1/2}, \text{ which gives } F_{TG}/F_{TA} = 0.794.$$

76. (a) We find the spring constant from energy conservation:

$$KE_i + PE_i = KE_f + PE_f;$$

$$\frac{1}{2}Mv_0^2 + 0 = 0 + \frac{1}{2}kA^2;$$

$$\frac{1}{2}(900 \text{ kg})(20 \text{ m/s})^2 = \frac{1}{2}k(5.0 \text{ m})^2, \text{ which gives } k = 1.4 \times 10^4 \text{ N/m.}$$

- (b) We find the period of the oscillation from

$$T = 2\pi(m/k)^{1/2} = 2\pi[(900 \text{ kg}) / (1.44 \times 10^4 \text{ N/m})]^{1/2} = 1.57 \text{ s.}$$

The car will be in contact with the spring for half a cycle, so the time is

$$t = \frac{1}{2}T = \frac{1}{2}(1.57 \text{ s}) = 0.79 \text{ s.}$$

77. (a) The speed of the wave in a string is $v = [F_T/\mu]^{1/2}$. Because the tensions must be the same anywhere along the string, for the ratio of velocities we have

$$v_2/v_1 = (\mu_1/\mu_2)^{1/2}.$$

- (b) Because the motion of one string is creating the motion of the other, the frequencies must be the same. For the ratio of wavelengths we have

$$\lambda_2/\lambda_1 = v_2/v_1 = (\mu_1/\mu_2)^{1/2}.$$

- (c) From the result for part (b) we see that, if $\mu_2 > \mu_1$, we have $\lambda_2 < \lambda_1$, so the **lighter cord** will have the greater wavelength.

78. The object will leave the surface when the maximum acceleration of the SHM becomes greater than g , so the normal force becomes zero. For the pebble to remain on the board, we have

$$a_{\max} = \omega^2 A = (2\pi f)^2 A < g;$$

$$[2\pi(3.5 \text{ Hz})]^2 A < 9.80 \text{ m/s}^2, \text{ which gives } A < 2.0 \times 10^{-2} \text{ m} = \text{2.0 cm}.$$

79. In the equilibrium position, the net force is zero, so we have

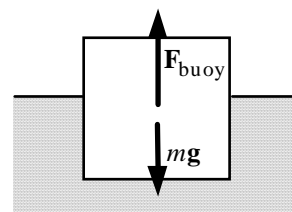
$$F_{\text{buoy}} = mg.$$

When the block is pushed into the water, there will be an additional buoyant force, equal to the weight of the additional water displaced, to bring the block back to the equilibrium position. When the block is pushed down a distance Δx , this net upward force is

$$F_{\text{net}} = -\rho_{\text{water}} g A \Delta x.$$

Because the net restoring force is proportional to the displacement, the block will oscillate with SHM. We find the effective force constant from the coefficient of Δx :

$$k = \rho_{\text{water}} g A.$$



80. The distance the mass falls is the distance the spring is stretched. We use energy conservation between the initial point, where the spring is unstretched, and the lowest point, our reference level for the gravitational potential energy, to find the spring constant:

$$KE_i + PE_i = KE_f + PE_f;$$

$$0 + mgh = 0 + \frac{1}{2}kh^2, \text{ which gives } k = 2mg/h.$$

We find the frequency from

$$f = (k/m)^{1/2}/2\pi = (2mg/hm)^{1/2}/2\pi = (2g/h)^{1/2}/2\pi = [2(9.80 \text{ m/s}^2)/(0.30 \text{ m})]^{1/2}/2\pi = \text{1.3 Hz}.$$

81. The increase in temperature will cause the length of the brass rod to increase. The period of the pendulum depends on the length,

$$T = 2\pi(L/g)^{1/2},$$

so the period will be greater. This means the pendulum will make fewer swings in a day, so the clock will be **slow**.

We use T_C for the temperature to distinguish it from the period.

For the length of the brass rod, we have

$$L = L_0(1 + \alpha \Delta T_C).$$

Thus the ratio of periods is

$$T/T_0 = (L/L_0)^{1/2} = (1 + \alpha \Delta T_C)^{1/2}.$$

Because $\alpha \Delta T_C$ is much less than 1, we have

$$T/T_0 \approx 1 + \frac{1}{2}\alpha \Delta T_C, \text{ or } \Delta T/T_0 = \frac{1}{2}\alpha \Delta T_C.$$

The number of swings in a time t is $N = t/T$. For the same time t , the change in period will cause a change in the number of swings:

$$\Delta N = (t/T) - (t/T_0) = t(T_0 - T)/TT_0 \approx -t(\Delta T/T_0)/T_0,$$

because $T \approx T_0$. The time difference in one day is

$$\begin{aligned} \Delta t &= T_0 \Delta N = -t(\Delta T/T_0) = -t\left(\frac{1}{2}\alpha \Delta T_C\right) \\ &= -(1 \text{ day})(86,400 \text{ s/day})\left[\frac{1}{2}(19 \times 10^{-6} \text{ (C}^\circ)^{-1})(35^\circ\text{C} - 20^\circ\text{C})\right] = -12.3 \text{ s.} \end{aligned}$$

82. When the water is displaced a distance Δx from equilibrium, the net restoring force is the unbalanced weight of water in the height $2\Delta x$:

$$F_{\text{net}} = -2\rho g A \Delta x.$$

We see that the net restoring force is proportional to the displacement, so **the block will oscillate with SHM**.

We find the effective spring constant from the coefficient of Δx :

$$k = 2\rho g A.$$

From the formula for k , we see that the effective spring constant depends on **the density and the cross section**.

